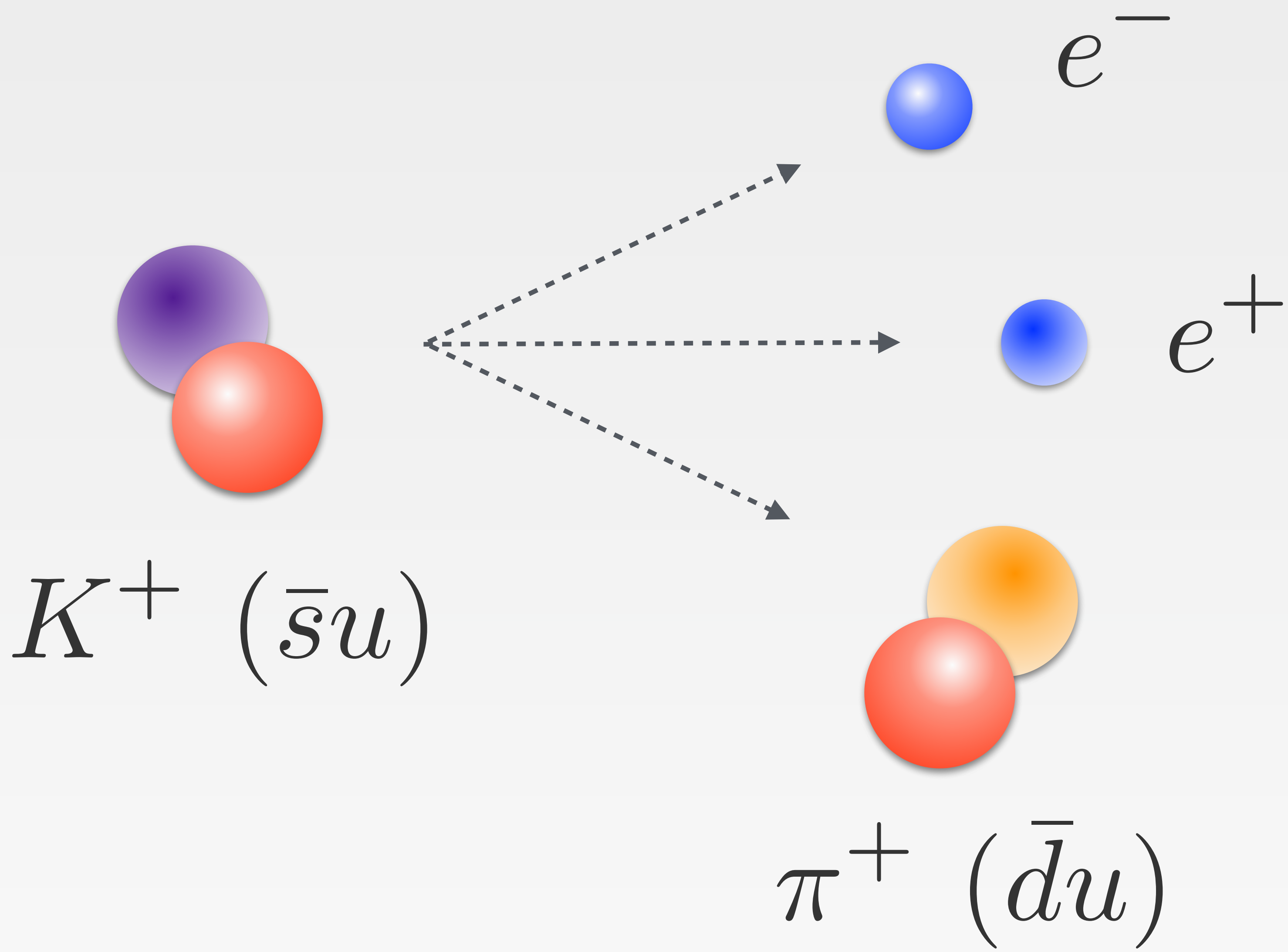

Proof-of-concept lattice calculations for rare kaon decay amplitudes

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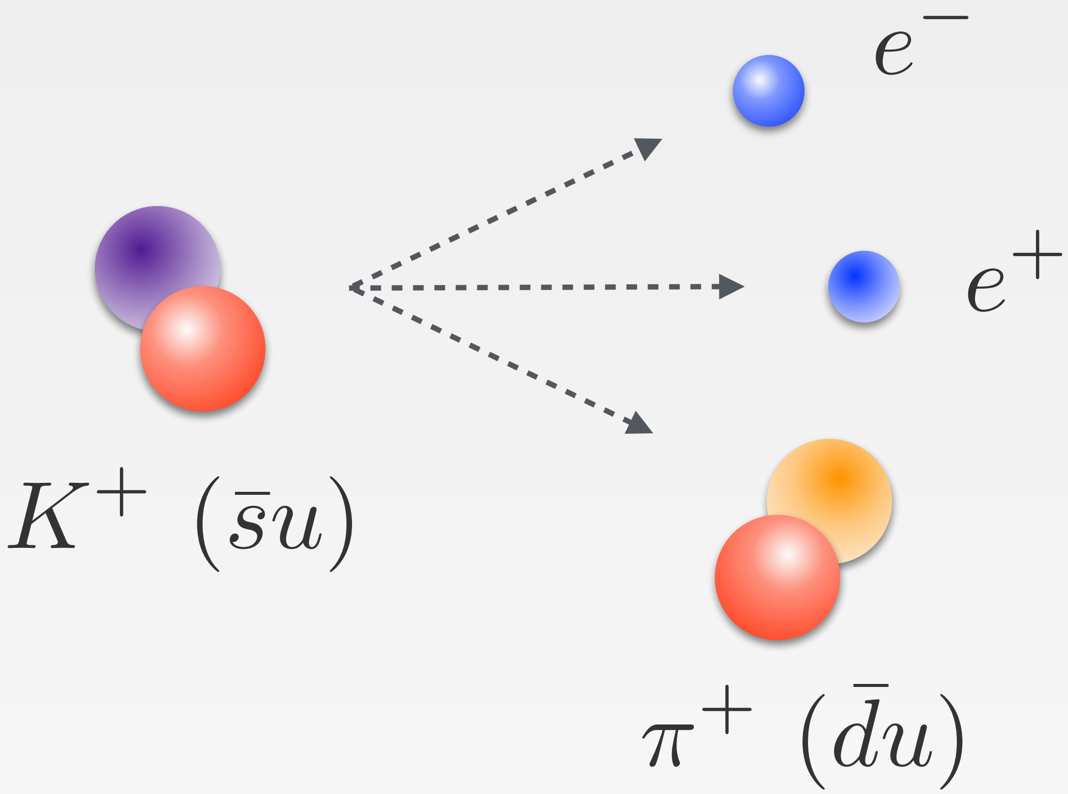
- Motivations
- $K \to \pi \ell^+ \ell^-$ decays
- $K^+ \to \pi^+ \bar{\nu} \nu$ decays
- Conclusion & perspectives

Motivations

Searching for new physics



Flavour Changing Neutral Current

Extremely rare in the SM

$\Rightarrow$ sensitive to new physics

Two new experiments **in progress** at CERN and J-PARC, **important results are expected in the next five years.**

Improved theory predictions are needed.

Decay channels

- $K^+ \to \pi^+ \ell^+ \ell^-$
  Long-distance dominated, "easy" to see experimentally.
- $K^0_{L/S} \to \pi^0 \ell^+ \ell^-$
  Long-distance dominated, interesting CP violations.
- $K^+ \to \pi^+ \bar{\nu} \nu$
  Mainly short-distance (top loop), **NA62 Run 1**.
  Long-distance charm effects?
- $K^0_{L/S} \to \pi^0 \bar{\nu} \nu$
  Short-distance (top loop) dominated. KOTO experiment.

Lattice

$K \to \pi \ell^+ \ell^-$ decays

Long-distance amplitude

EM current

$$\mathscr{A}_\mu^c(q^2) = \int \mathrm{d}^4x \, \langle \pi^c(\mathbf{p}) | \, \mathrm{T}[J_\mu(0) H_W(x)] \, | K^c(\mathbf{k}) \rangle$$

$\Delta S = 1$ Effective weak Hamiltonian

$$\mathscr{A}_\mu^c(q^2) = -i \frac{G_F}{(4\pi)^2} [q^2 (k+p)_\mu - (M_K^2 - M_\pi^2) q_\mu] V_c(z)$$

$$V_c(z) = a_c + b_c z + V_c^{\pi\pi}(z) \qquad z = q^2 / M_K^2$$

SM prediction?

Lattice approach

- Lattice QCD: Monte-Carlo estimation of the full QCD Euclidean path integral. **Non-perturbative**.

- Challenge here: how to relate the decay amplitude to an Euclidean correlation function.

Minkowski spectral representation

$$\mathscr{A}_\mu^c(q^2) = i\int_0^{+\infty} \mathrm{d}E\, \frac{\rho(E)}{2E} \frac{\langle \pi^c(\mathbf{p})|\, J_\mu \,|E,\mathbf{k}\rangle \langle E,\mathbf{k}|\, H_W \,|K^c(\mathbf{k})\rangle}{E_K(\mathbf{k}) - E + i\varepsilon}$$
$$- i\int_0^{+\infty} \mathrm{d}E\, \frac{\rho_S(E)}{2E} \frac{\langle \pi^c(\mathbf{p})|\, H_W \,|E,\mathbf{p}\rangle \langle E,\mathbf{p}|\, J_\mu \,|K^c(\mathbf{k})\rangle}{E - E_\pi(\mathbf{p}) + i\varepsilon}$$

[RBC-UKQCD, PRD 92(9), 094512, 2015]

Euclidean spectral representation

$$\mathscr{A}_\mu^c(q^2, T_a, T_b) = -\int_0^{+\infty} \mathrm{d}E\, \frac{\rho(E)}{2E} \frac{\langle \pi^c(\mathbf{p})|\, J_\mu \,|E, \mathbf{k}\rangle \langle E, \mathbf{k}|\, H_W \,|K^c(\mathbf{k})\rangle}{E_K(\mathbf{k}) - E} \times (1 - e^{[E_K(\mathbf{k})-E]T_a}) + \int_0^{+\infty} \mathrm{d}E\, \frac{\rho_S(E)}{2E} \frac{\langle \pi^c(\mathbf{p})|\, H_W \,|E, \mathbf{p}\rangle \langle E, \mathbf{p}|\, J_\mu \,|K^c(\mathbf{k})\rangle}{E - E_\pi(\mathbf{p})} \times (1 - e^{-[E-E_\pi(\mathbf{p})]T_b})$$

Time integration range: $[-T_a, T_b]$.

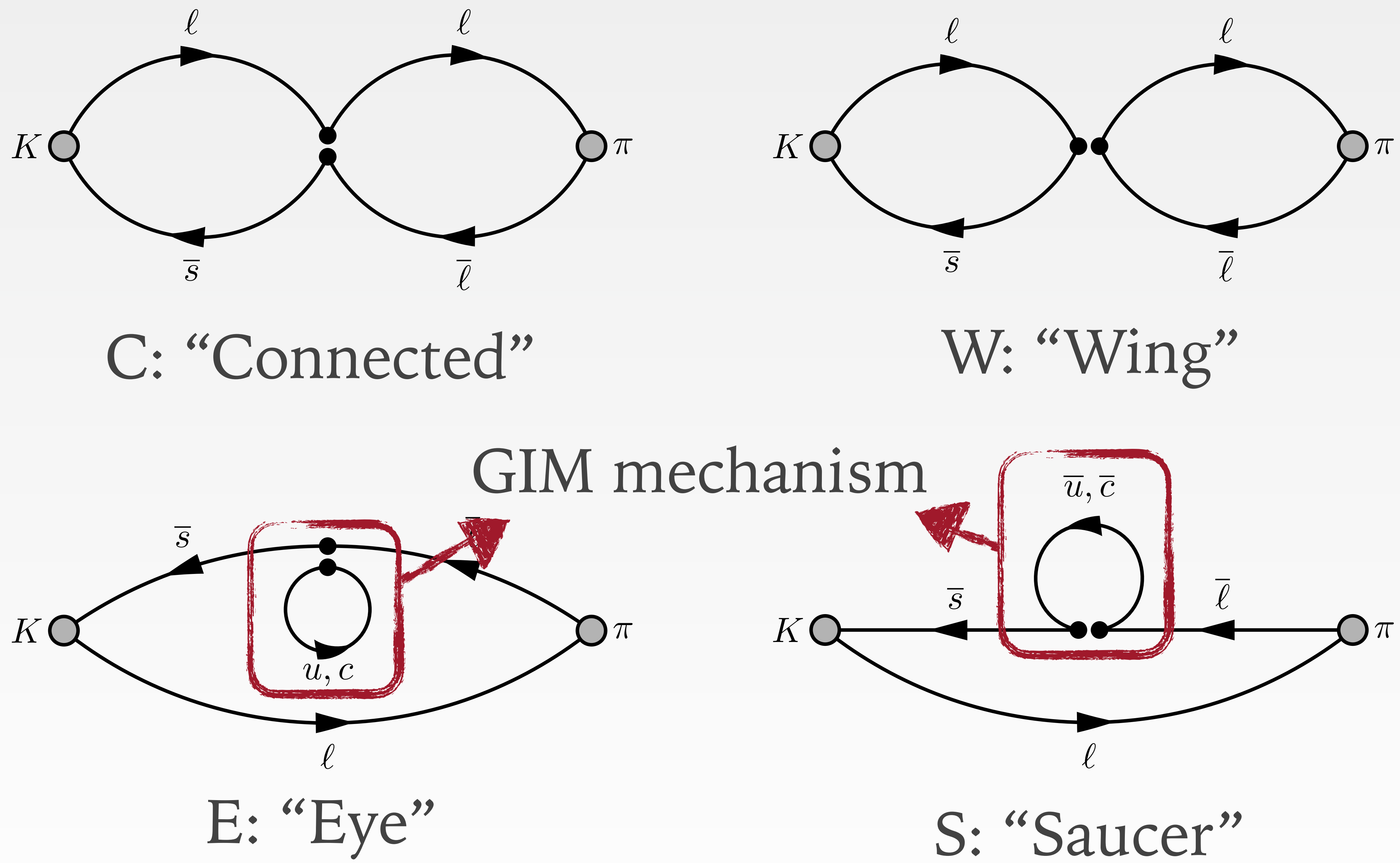
Diverges at infinite time for $E < E_K(\mathbf{k})$.

"Simple" here (only π, $\pi\pi\pi$).

Try to think about rare B decays!

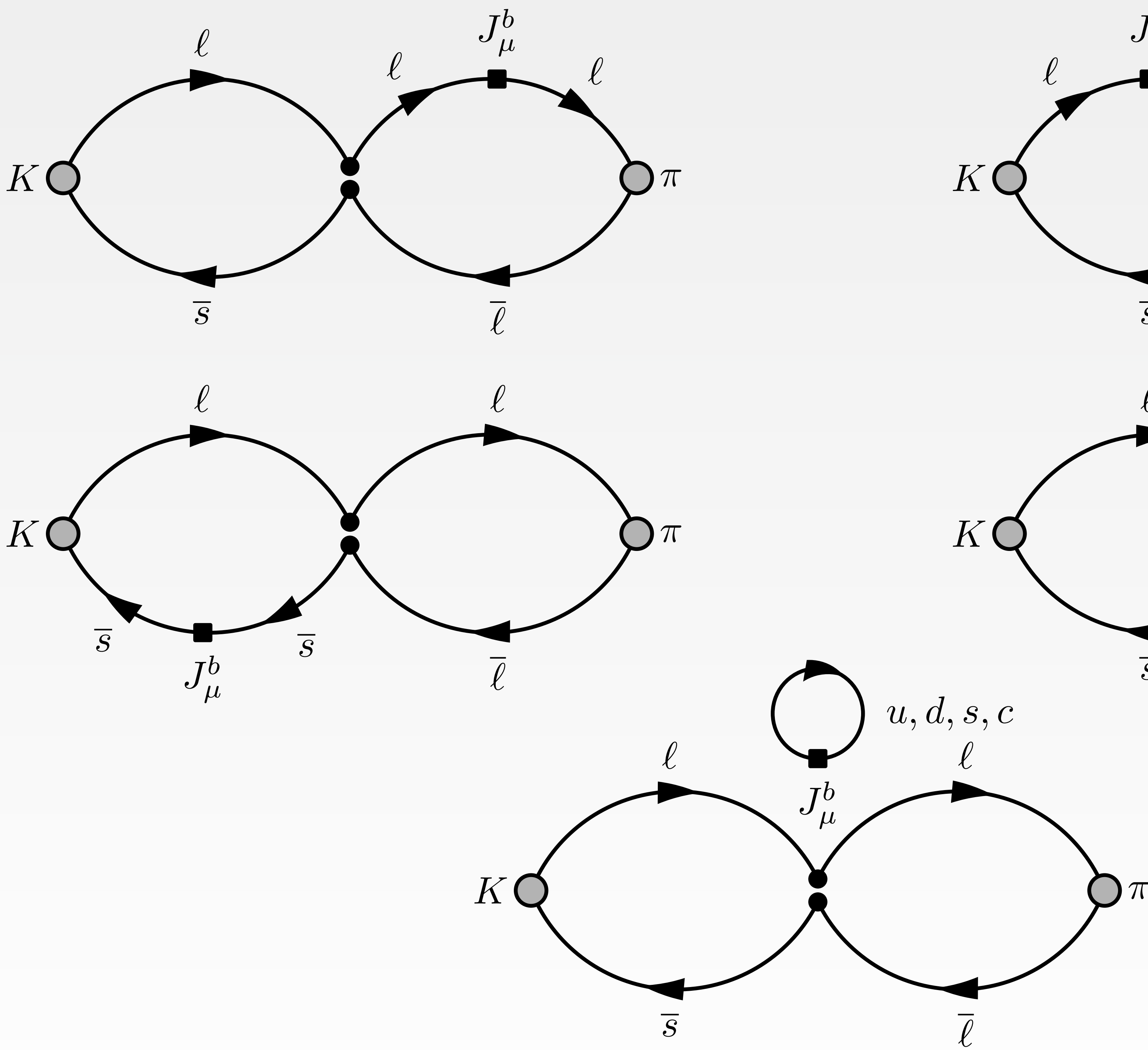
[RBC-UKQCD, PRD 92(9), 094512, 2015]

Lattice correlators



[RBC-UKQCD, PRD 92(9), 094512, 2015]

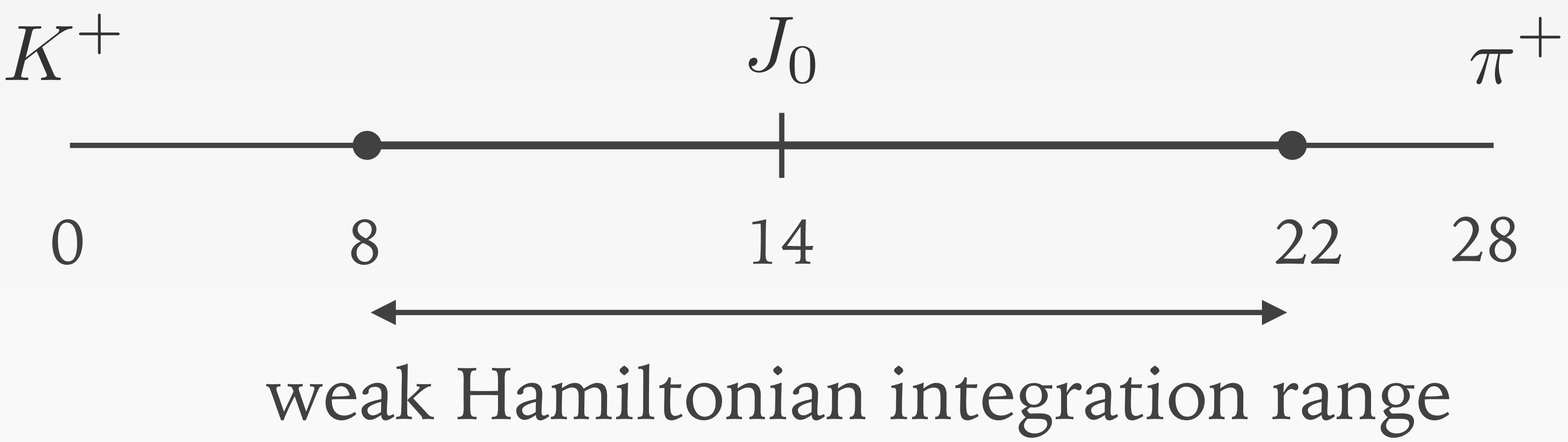
Lattice correlators



[RBC-UKQCD, PRD 92(9), 094512, 2015]

Lattice setup

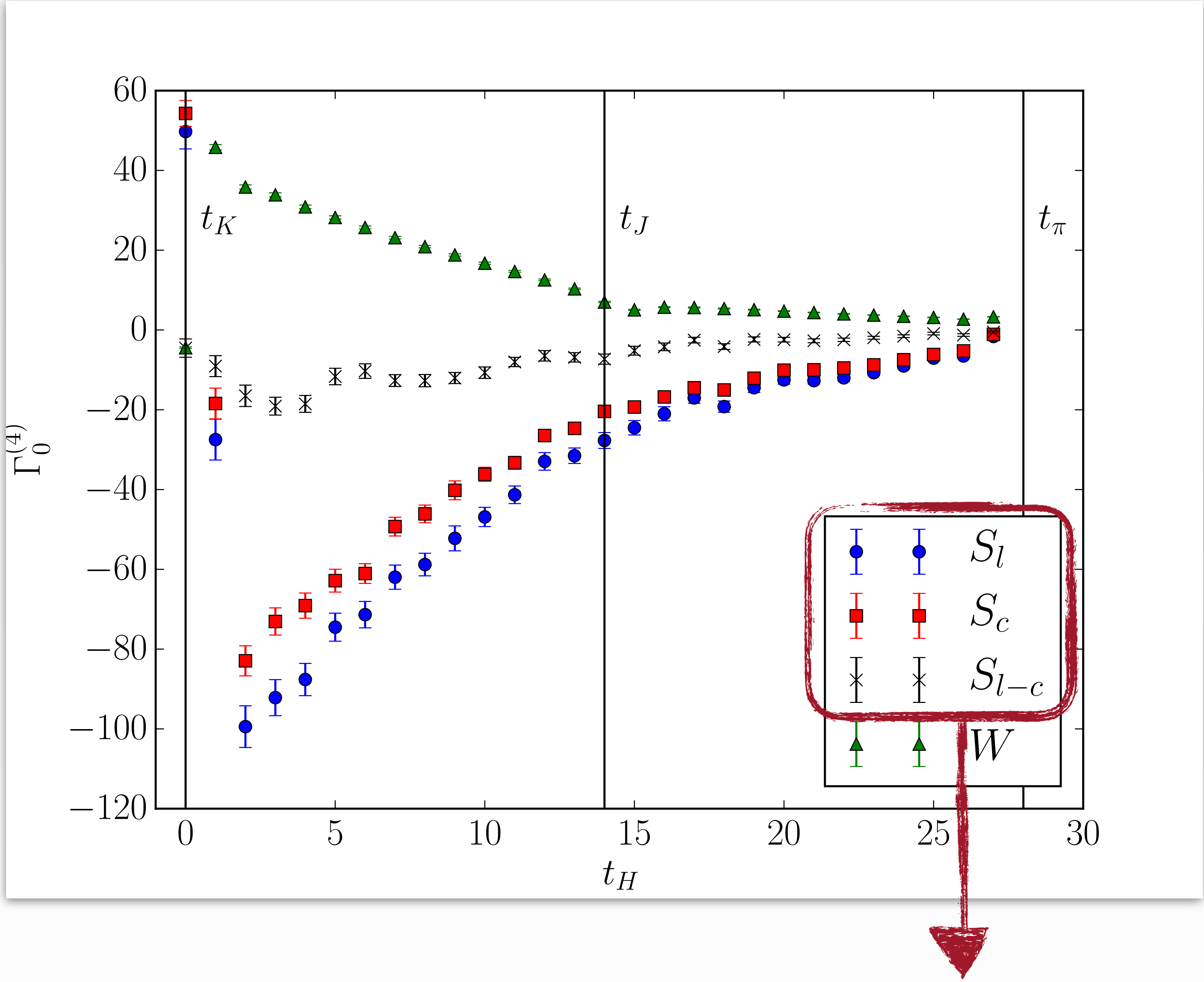
- DWF action, $24^3 \times 64$ lattice with spacing ~0.12 fm.
- $N_f = 2 + 1$, $M_\pi \simeq 420$ MeV and $M_K \simeq 600$ MeV .



- For this kinematics only single π state is problematic.

[RBC-UKQCD, PRD 94(1), 114516, 2016]

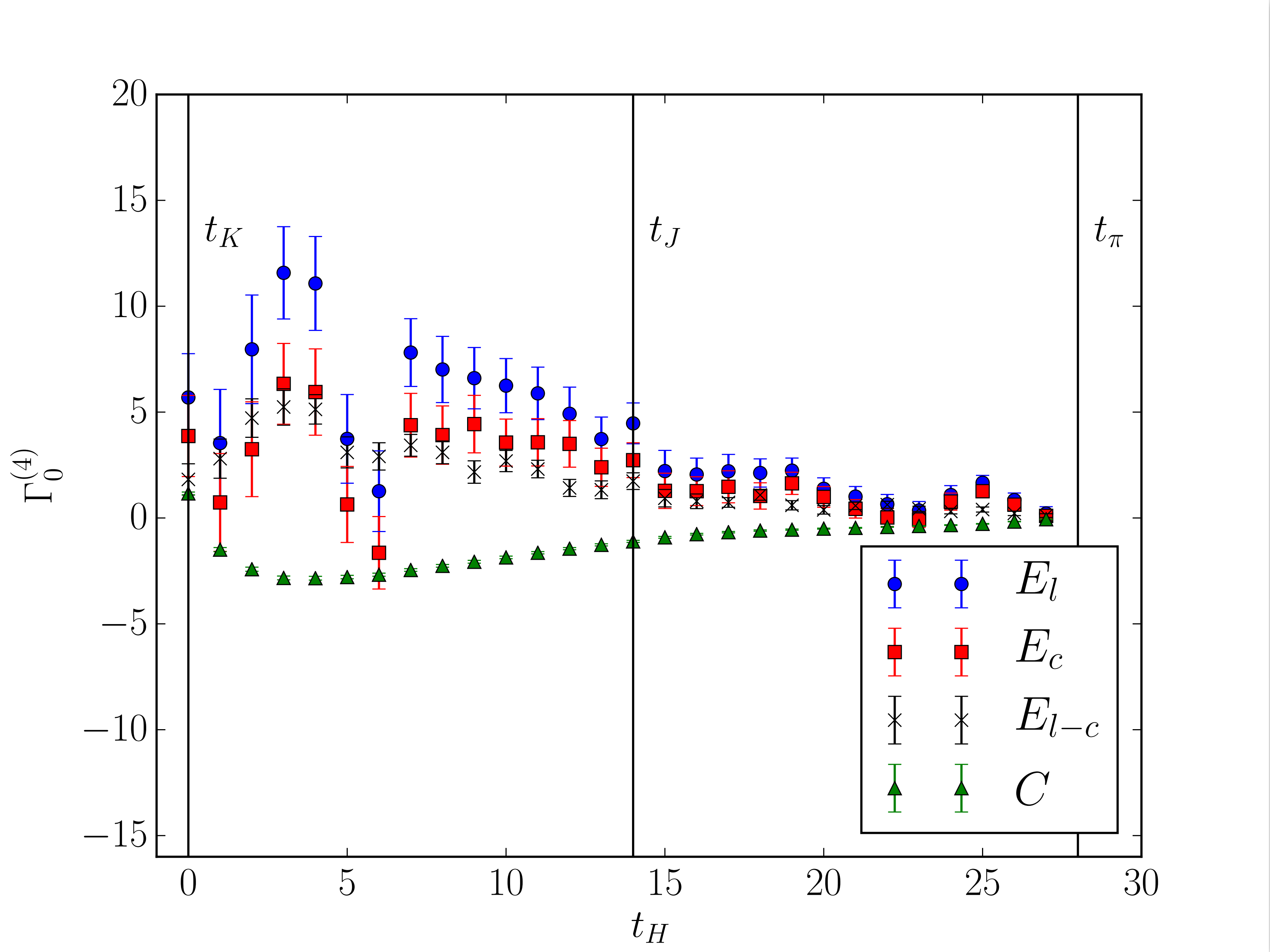
Results: correlators



GIM mechanism

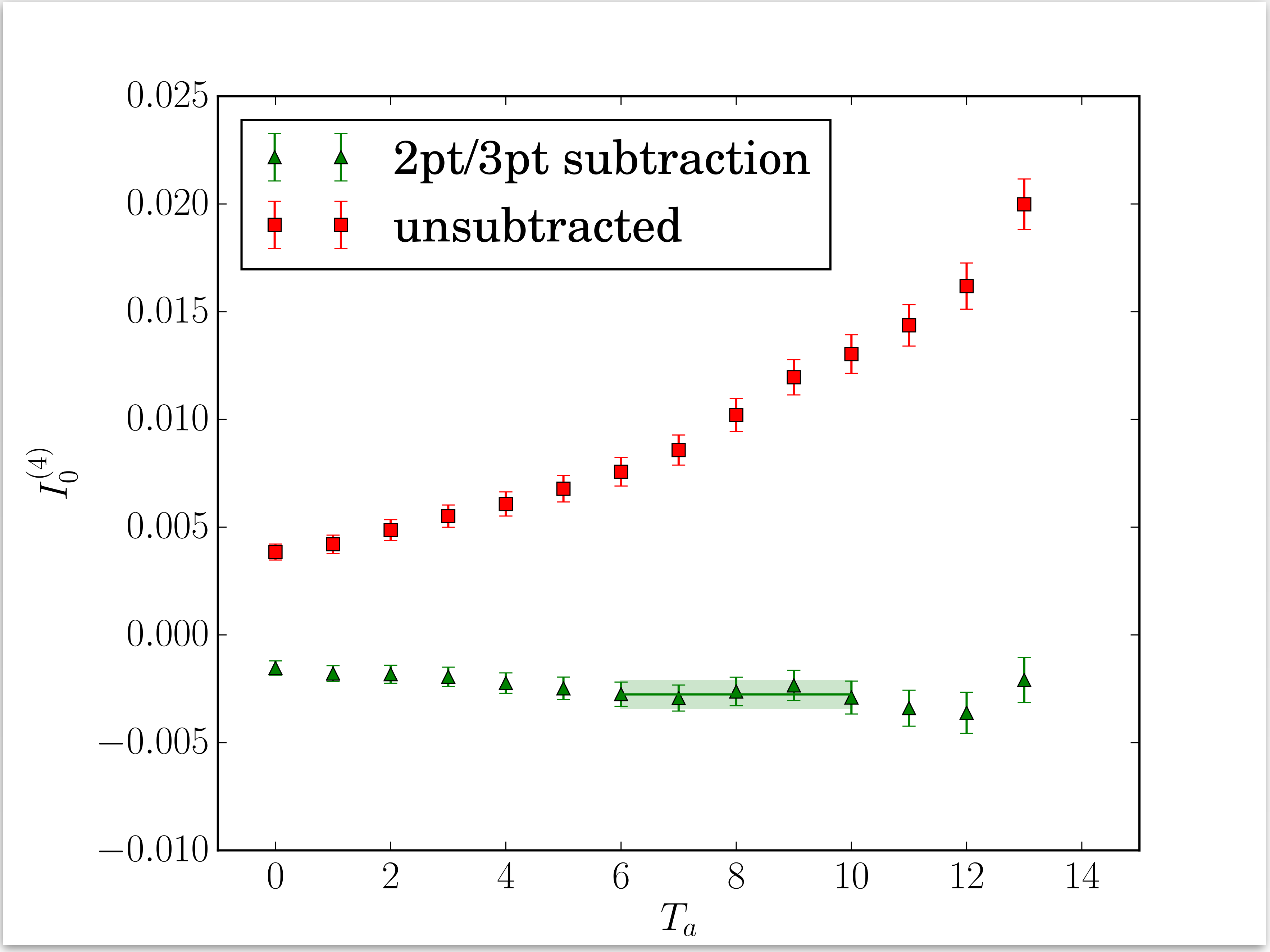
[RBC-UKQCD, PRD 94(1), 114516, 2016]

Results: correlators



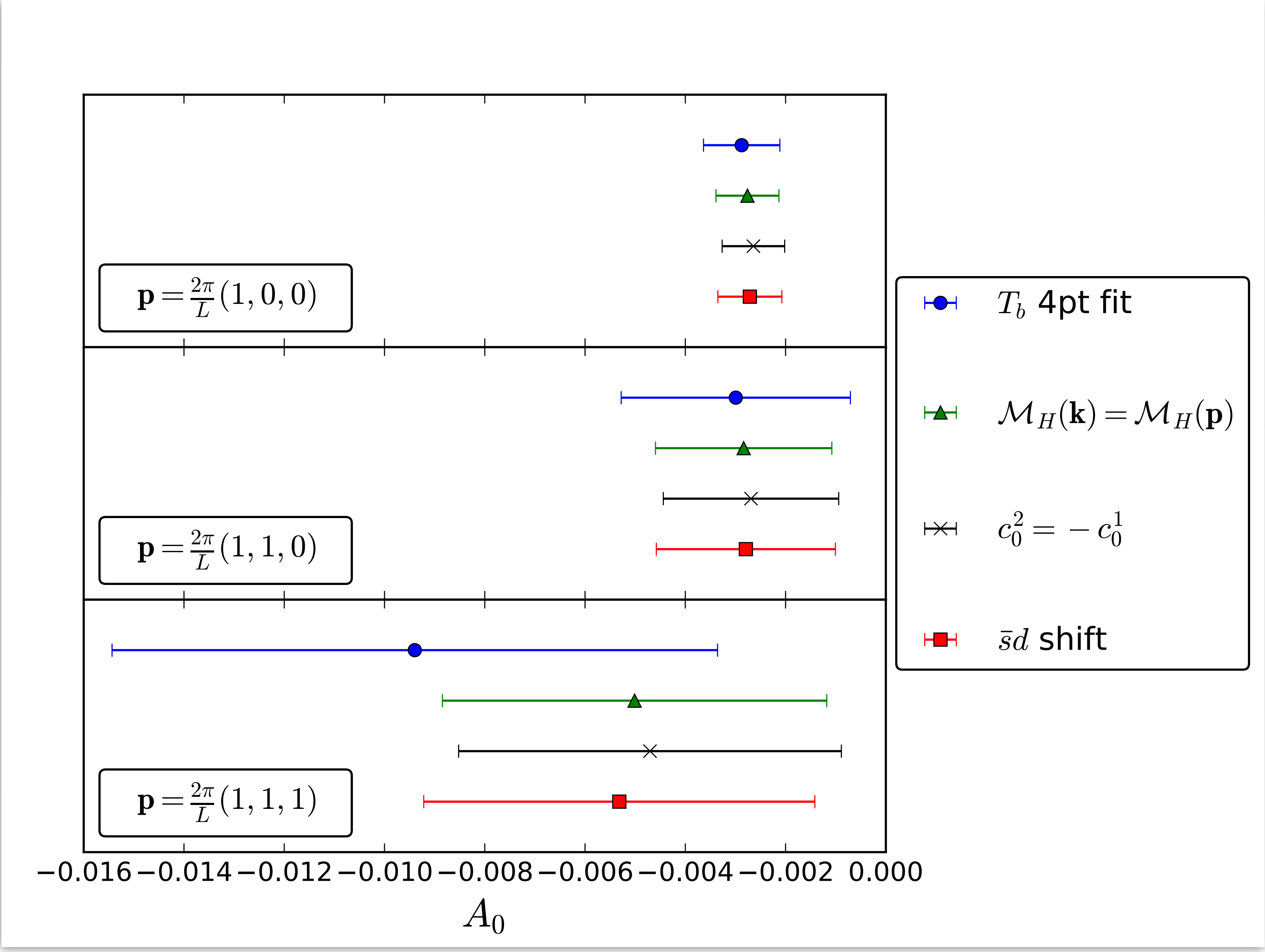
[RBC-UKQCD, PRD 94(1), 114516, 2016]

Results: exponential subtraction



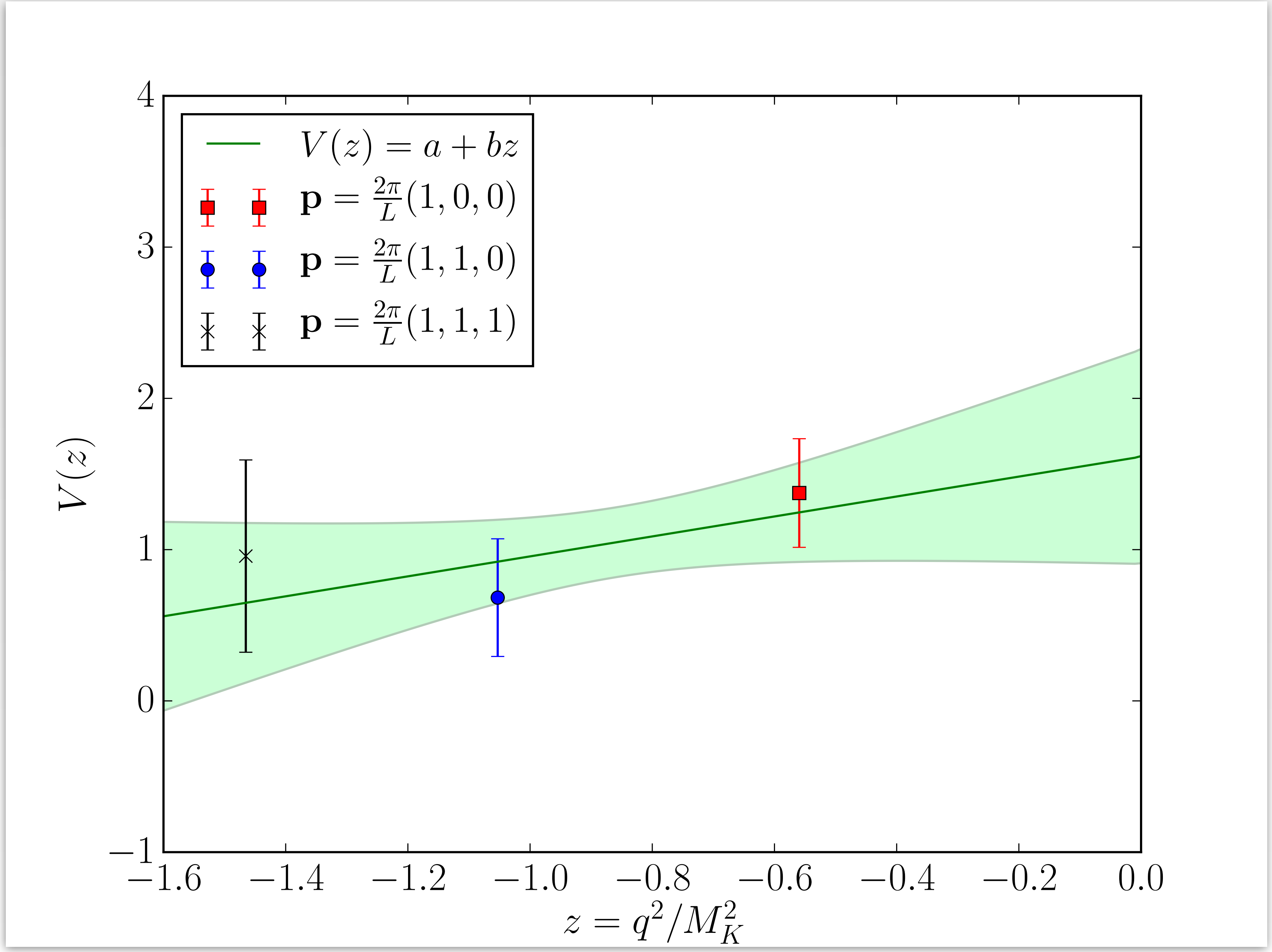
[RBC-UKQCD, PRD 94(1), 114516, 2016]

Results: exponential subtraction



[RBC-UKQCD, PRD 94(1), 114516, 2016]

Results: form factor



[RBC-UKQCD, PRD 94(1), 114516, 2016]

$K^+ \to \pi^+ \bar{\nu} \nu$ decays

$K \to \pi\bar{\nu}\nu$ branching ratio

$$\mathrm{Br}(K^+ \to \pi^+\bar{\nu}\nu) = \kappa \left\{ \left[\frac{\Im\lambda_t}{\lambda^5} X_t\left(\frac{m_t^2}{M_W^2}\right)\right]^2 + \left[\frac{\Re\lambda_c}{\lambda} P_c + \frac{\Re\lambda_t}{\lambda^5} X_t\left(\frac{m_t^2}{M_W^2}\right)\right]^2 \right\}$$

$$= 9.11(72) \times 10^{-11}$$

[Buras *et al.*, arXiv:1503.02693]

Top domination: $\sim$68%

Charm-up contribution: $\sim$32%

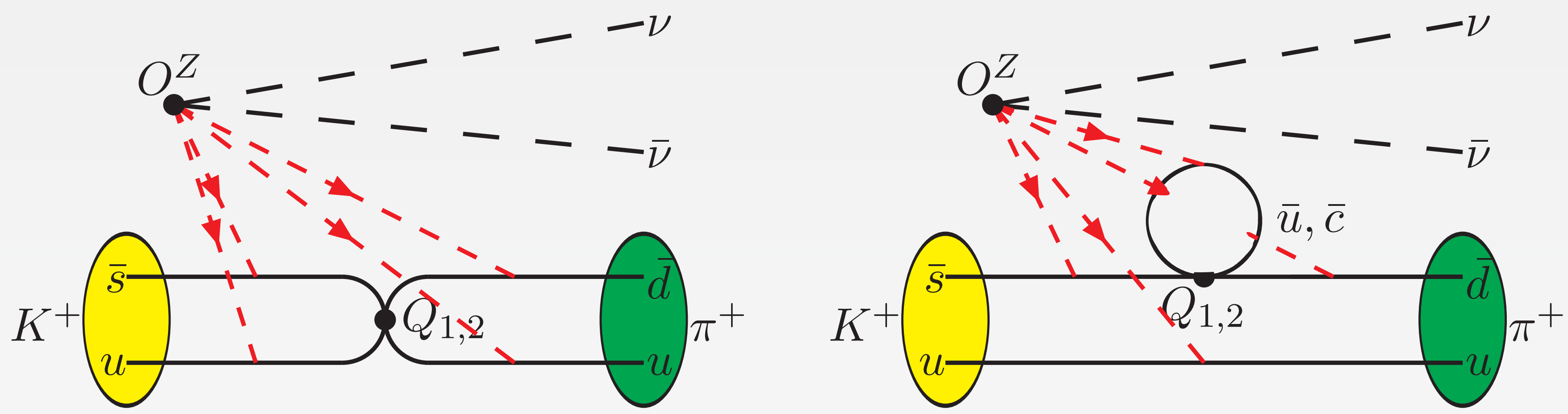
Short-distance: $\sim$29%

Long-distance: $\sim$3%

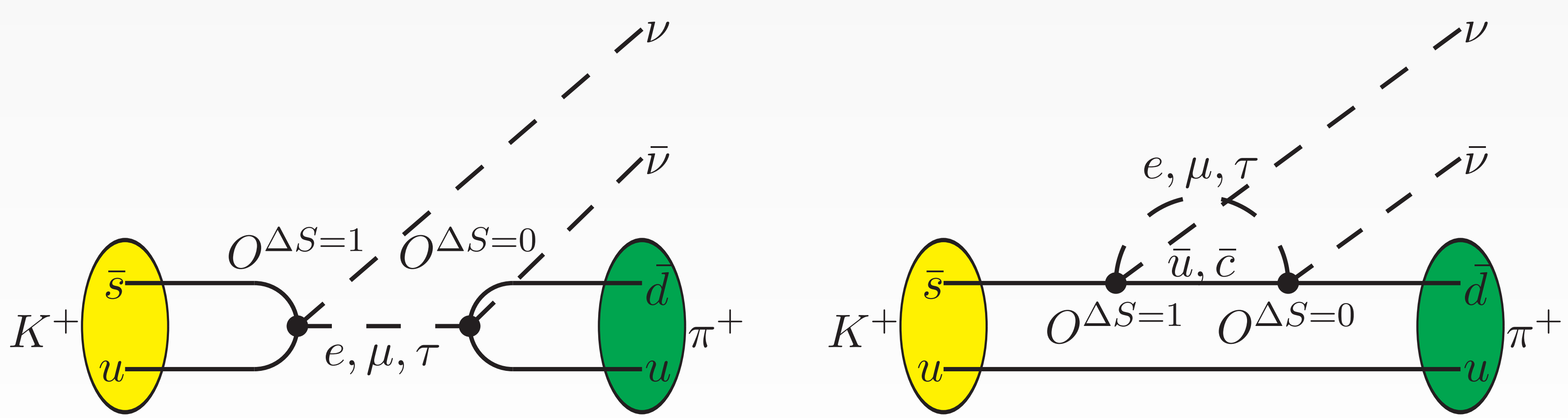
LD: significant source of uncertainty,
needs to be consolidated for NA62 results.

Long-distance amplitude

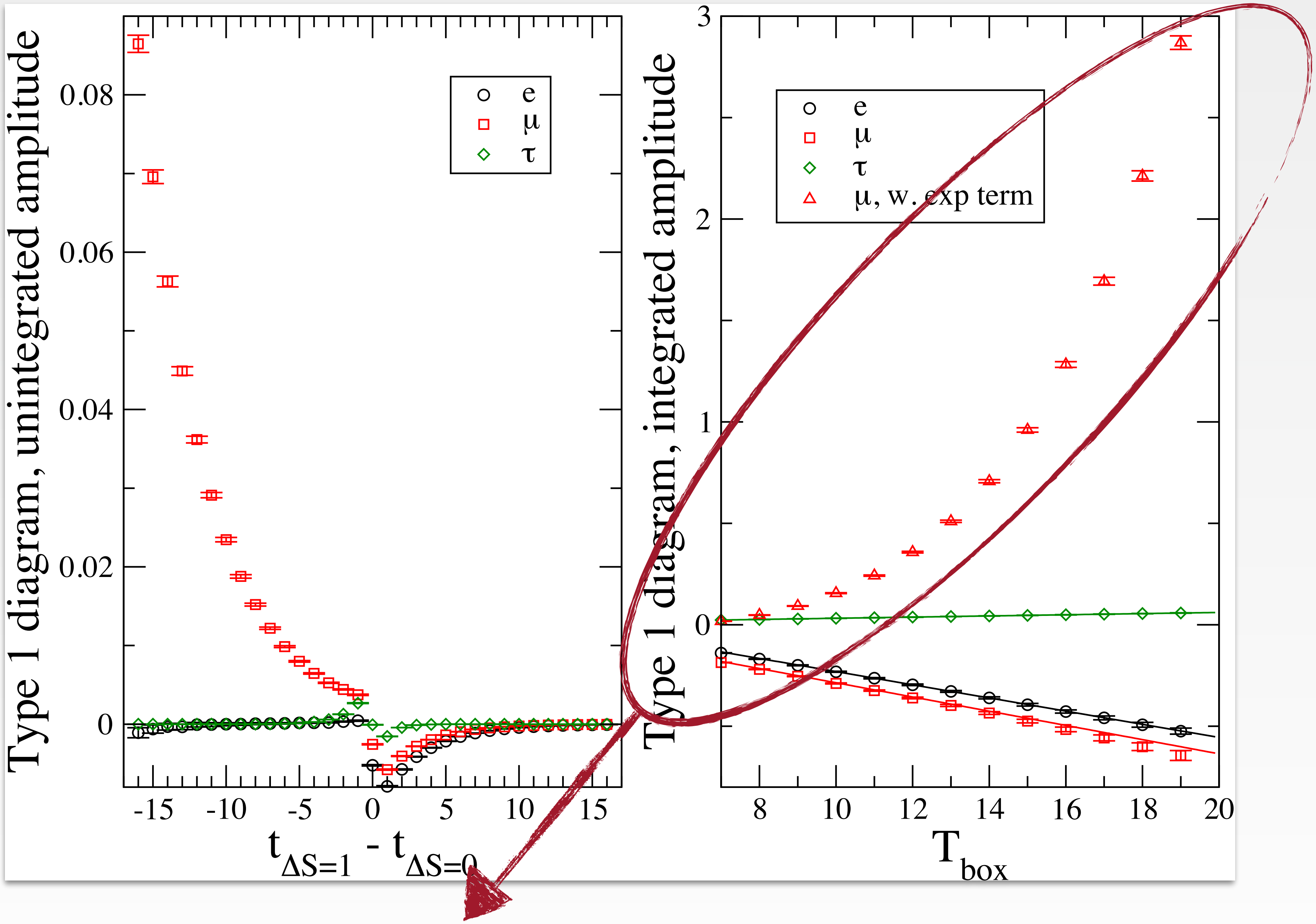
Same as $K \to \pi \ell^+ \ell^-$ with neutral weak current:



New: W-box diagrams:

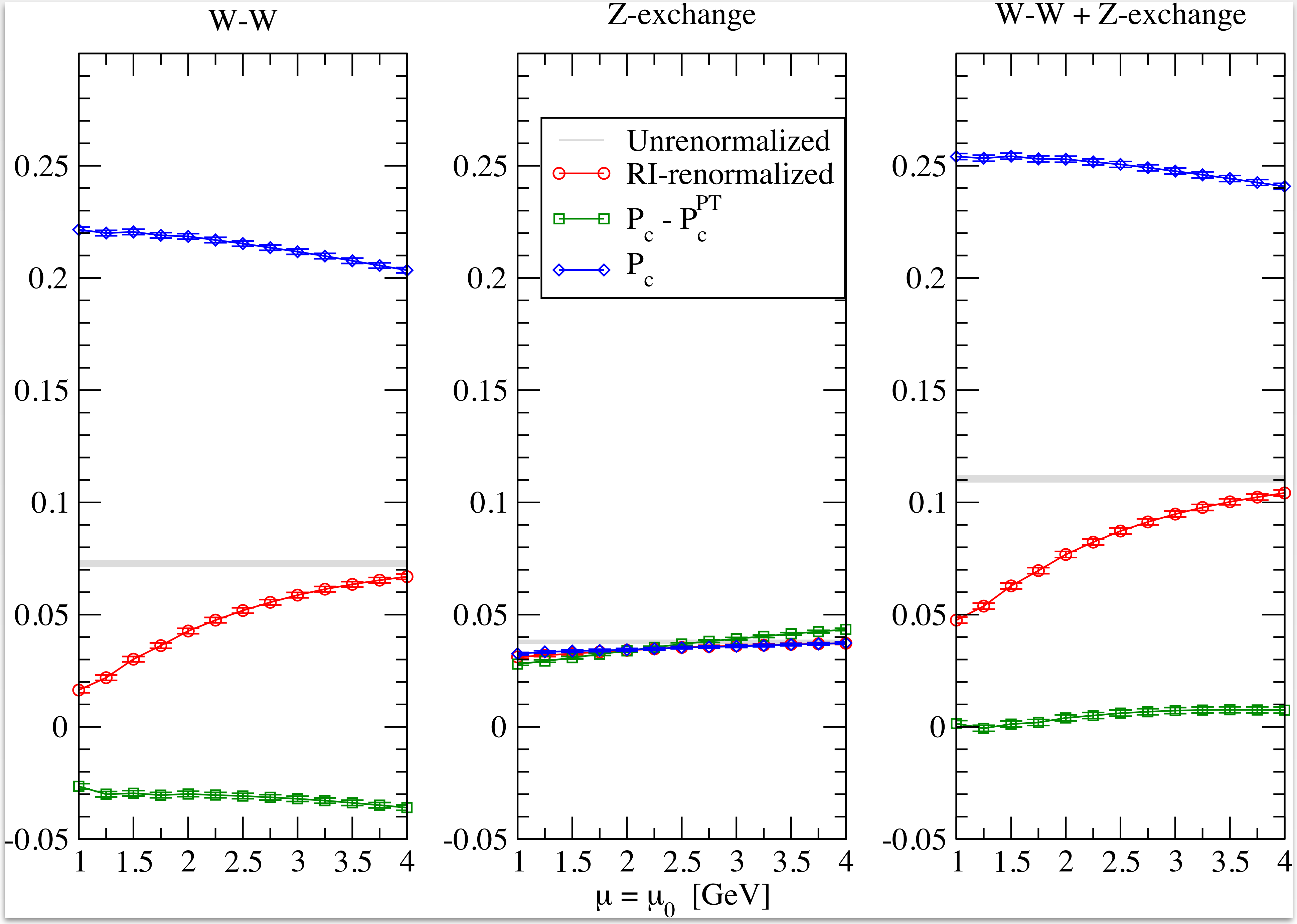


Analytical continuation issues



$|\mu^+\nu_\mu\rangle$ and $|\pi^0\mu^+\nu_\mu\rangle$ contamination

Charm contribution results



[RBC-UKQCD, PRL 118(2), 252001, 2017]

Conclusion & perspectives

Conclusion

- Lattice framework for rare K decays achieved.
- Proof-of-concept calculations successful.
- Results comparison with phenomenology/experiment difficult because of unphysical parameters.
- What I have not talked about: renormalisation. Quite involved, maybe still room for improvement.

Perspectives

- Physical quark calculation: now (**Fionn's talk**)! New Grid & Hadrons based code.
- Tesseract: new 35232 Xeon cores HPE SGI-8600 supercomputer in Edinburgh.
- $\pi\pi$ & $\pi\pi\pi$ contamination problematic?
- We are excited with the NA62 $K^+ \to \pi^+ \bar{\nu}\nu$ and $K^+ \to \pi^+ \mu^+ \mu^-$ results.
- $K^+ \to \pi^+ e^+ e^-$ in future runs?

Thank you!