Applications of the All-to-All method in a Lattice Calculation of the K->pill Amplitude.

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Overview of the Presentation

- Extracting observables from the lattice.
- Lattice propagators.
    - Point and wall source propagators
    - All-to-All propagators
- $K \to \pi\ell^+\ell^-$ on the lattice
- Exploratory 24^3 A2A results
- Physical point 48^3 gauge fixed wall source results

Correlation Functions on the Lattice

If, for a 2-point function we insert a complete set of states, ignore the “around-the-world” effects and use the Schrödinger picture

$$\tilde{C}_\pi(\boldsymbol{p}, t, t_\pi) = \sum_n \frac{|A_\pi(\boldsymbol{p})|^2}{2E_n(\boldsymbol{p})} e^{-E_n(\boldsymbol{p})(t-t_\pi)}$$

For $t >> t_\pi$ the ground state will dominate and so the correlation function can be fit to extract the parameters $E_\pi(\boldsymbol{p})$ and $A_\pi(\boldsymbol{p}) = \left| \langle 0 | \tilde{O}_\pi(\boldsymbol{p}) | \pi \rangle \right|$.

Computing propagators

$S^q(x, y)$ is given by the inverse of the Dirac matrix for that quark

- for each of the 12 spin/color combinations
- describes the propagation amplitude from each of the N lattice sites to every other site
- with each element being a complex number
- for near singular matrices

Performing a full inversion of the Dirac matrix is for each gauge configuration is not practical, and typically translational invariance is taken advantage of to fix $y = y_0$.

Point and Wall Source Propagators

$S(x, y_0)$ can be found by solving the equation

$$S(x, y_0) = D^{-1}(x, y)\delta(y - y_0).$$

Another choice is a wall source:

$$\eta_W(y, t_0, \boldsymbol{p}) = \delta(t_y - t_0)e^{i\boldsymbol{p}\cdot\boldsymbol{y}}$$

which gives the sum of all point sources over the timeslice t_0:

$$S(x, y_0) = \sum_{\boldsymbol{y}} D^{-1}(x, y)\eta_W(y, t_0, \boldsymbol{p})$$

All-to-All Propagators

An approximation of $D^{-1}(x, y)$ where the eigenvalues and eigenvectors of D, λ_i and ϕ_i, are used to solve the low modes exactly. For a set of random noise source vectors that have the property

$$\lim_{N_h \to \infty} \sum_{h=1}^{h=N_h} \eta_h \eta_h^\dagger = \mathbb{1},$$

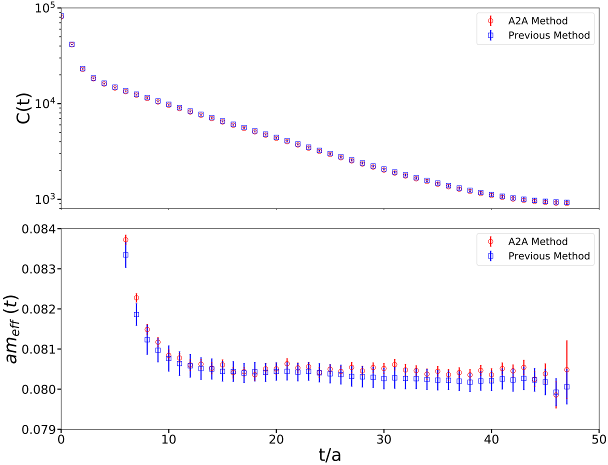
and defining

$$D_{defl}^{-1} = D^{-1} - \sum_{j=1}^{j=N_l} \frac{\phi_j \phi_j^\dagger}{\lambda_j},$$

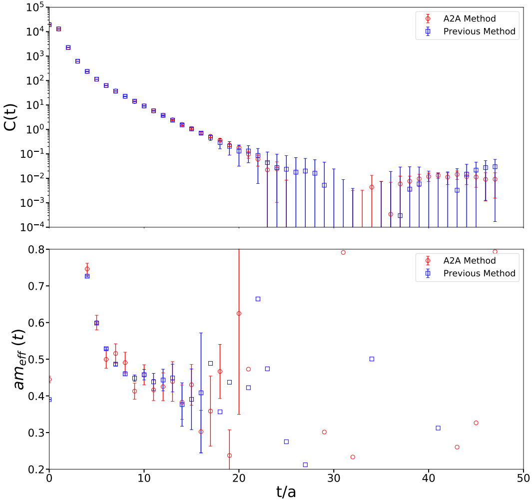
we can write

$$D_{A2A}^{-1} = \sum_{i=1}^{i=N_l} \frac{\phi_i \phi_i^\dagger}{\lambda_i} + \sum_{h=1}^{h=N_h} D_{defl}^{-1} \eta_h \eta_h^\dagger$$

A2A Pion 2 Point Function



A2A Vector 2 Point Function

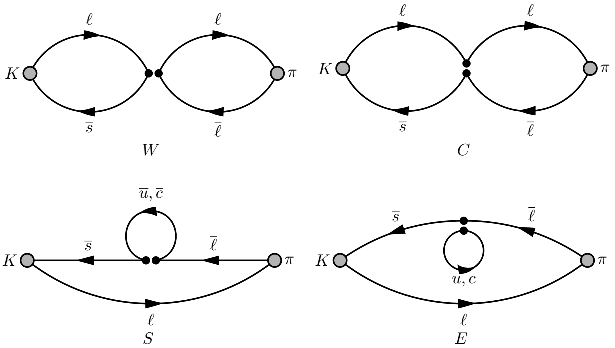


$K \to \pi$ 3-pt Function

To study $K \to \pi\ell^+\ell^-$ we consider the 3-point function:

$$\Gamma_H^{(3)}(t_H, \mathbf{p}) = \int d^3\mathbf{x} \left\langle \phi_\pi(t_\pi, \mathbf{p}) H_W(t_H, \mathbf{x}) \phi_K^\dagger(0, \mathbf{p}) \right\rangle$$

where $t_K < t_H < t_\pi$ and the integral over $\mathbf{x}$ is replaced by the sum over the discrete lattice.

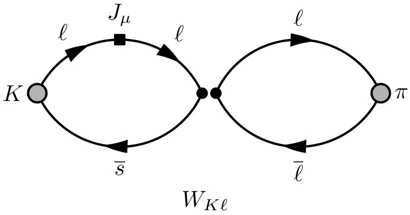


Current Insertion

The full decay amplitude also requires the insertion of an electromagnetic current:

$$\Gamma^{(4)}_{\mu}(t_H, t_J, \mathbf{k}, \mathbf{p}) = \int \mathrm{d}^3\mathbf{x} \int \mathrm{d}^3\mathbf{y}\, e^{-i\mathbf{q}\cdot p\mathbf{x}} \left\langle \phi_{\pi^j}(t_\pi, \mathbf{p}) T[J_\mu(t_J, \mathbf{x}) H_W(t_H, \mathbf{y})]\, \phi^\dagger_K(0, \mathbf{k}) \right\rangle ,$$

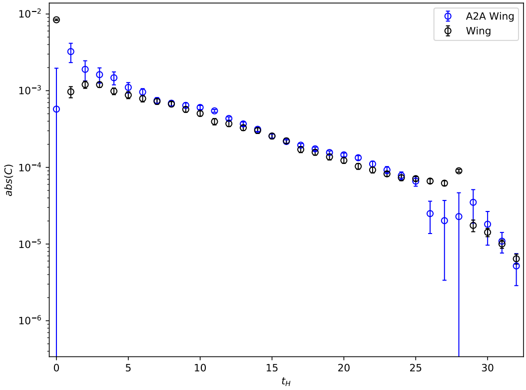
where $t_K < t_J, t_H < t_\pi$.



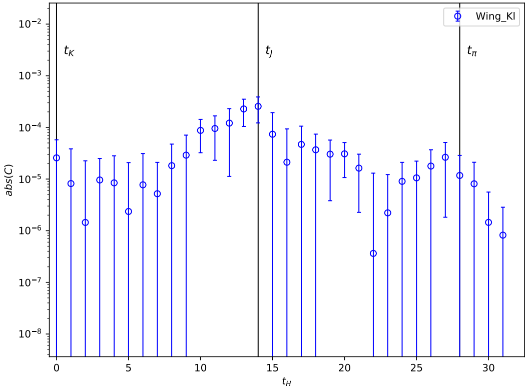
Exploratory 24^3 Setup

- $24^3 \times 64$ gauge configuration, $M_\pi \approx 420$MeV, $M_K \approx 600$MeV
- Domain Wall Fermion action
- Degenerate light quarks
- $am_l = 0.05$, $am_s = 0.2$, $M_5 = 1.8$, $Ls = 16$
- $t_K = 0$, $t_J = 14$, $t_\pi = 28$

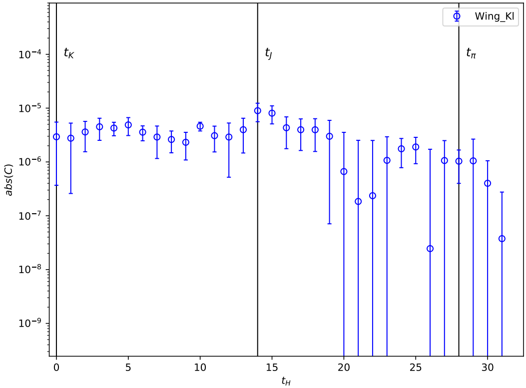
3 Point Function 24^3 Result



A2A Stochastic Current Insertion 24^3 Result



A2A Sequential Current Insertion 24^3 Result



Approach for Physical Point Calculation

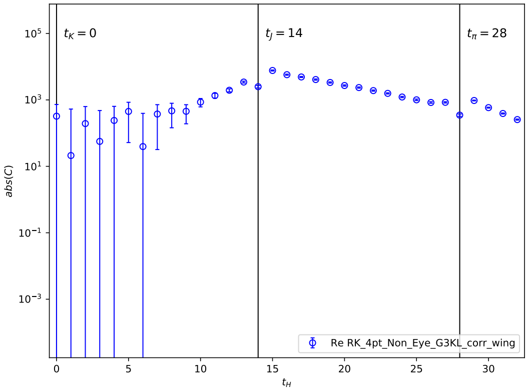
- Revert to wall sources for "non-eye" diagrams, with Coulomb gauge fixing
- Combine a mix of wall source propagators and A2A propagators for "eye" diagrams
  - Eye diagrams are usually noisier because of the $S^q(x_H, x_H)$ loop
  - A2A propagators are well suited to solve this problem

We have already started to gather statistics for the non-eye diagrams with this approach.

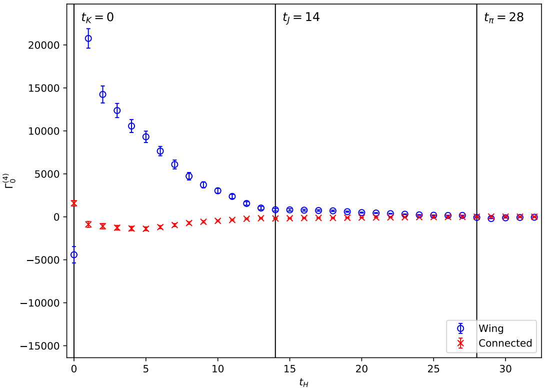
Physical Point Setup

- $48^3 \times 96$ gauge configuration, $m_\pi \approx 140$ MeV, $m_K \approx 500$ MeV
- Degenerate light quarks
- Scaled DWF action for strange quark
- $am_s = 0.0362$, $M5 = 1.8$, $Ls = 24$, $scale = 2.0$
- ZMöbius DWF for light quarks
- $am_l = 0.00078$, $M_5 = 1.8$, $Ls = 10$
- $t_K = 0$, $t_J = 14$, $t_\pi = 28$
- $\mathbf{p}_K = (0, 0, 0)$, $\mathbf{p}_\pi = \frac{2\pi}{L}(1, 0, 0)$,

4 Point Function Sequential Current Insertion 48^3 Result



Combining All Non-Eye Diagrams: 48^3 Result



Appropriate fractional quark charges and Wilson coefficients have been applied, though there is a missing volume factor.

Remaining Challenges

The remaining computational challenges are related to the loop in the eye diagrams.

- Current insertion on the loop
- Charm quark loop
    - Requires an extrapolation to physical the mass in order to use the same discretized action

Summary

- A2A vectors are a powerful tool for approximating quark propagators
  - But the are not always appropriate to use for a full calculation
  - A2A propagators will be used to supplement the $K \to \pi\ell^+\ell^-$ calculation
- Physical point simulations have already begun

Thank you.