Useful Entanglement for Quantum Technologies

Augusto Smerzi
QSTAR, INO-CNR & LENS
Firenze, Italy

INO-CNR

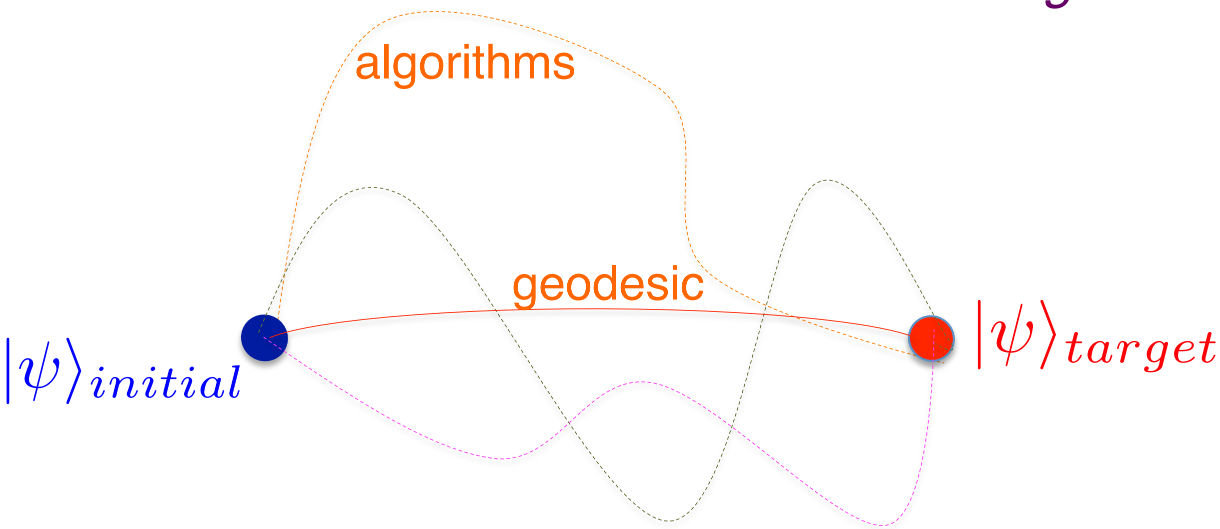
QSTAR
Quantum Science and Technology in ARcetri

Lens
European Laboratory for Non-Linear Spectroscopy

What is a QT?

Examples: quantum computation,
Grover search algorithm, …

Sufficient condition:
entanglement enhanced performances



Holy grail:
Move from the initial state (preparation) to the target state (solution of the task) along a geodesic and as fast as possible with $|\langle\psi_{initial}|\psi_{target}\rangle| \sim 0$

To quantify, we need to introduce a "distance" and a "velocity" among quantum probability distributions

Distance between probabilities:

In analogy with distances among points in space
it measures how far away (how much different) are two probability distributions

$$D[p(x), q(x)] \geq 0$$

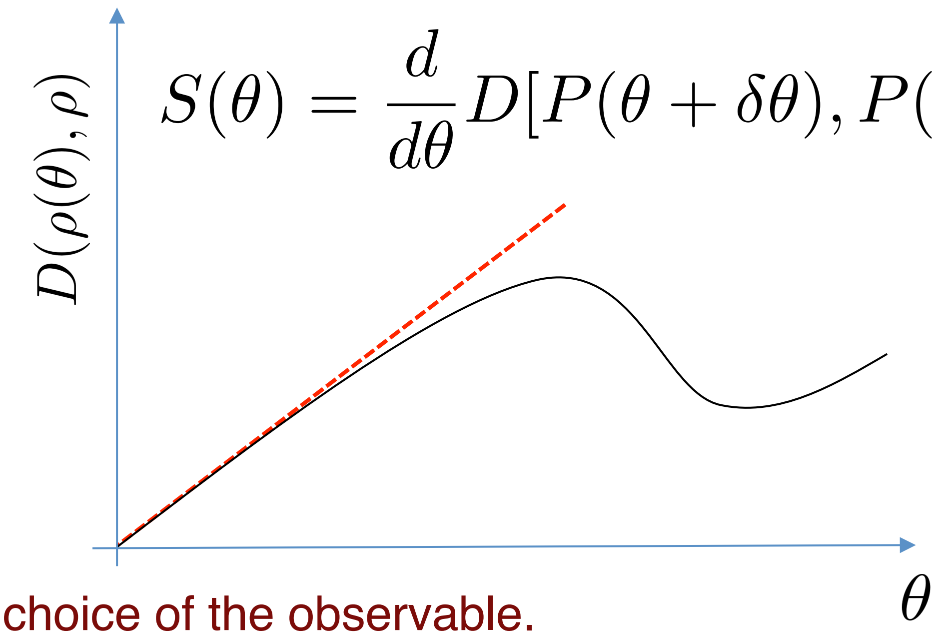
$$D[p(x), q(x)] + D(q(x), r(x)] \geq D[q(x), r(x)]$$

and speed

$$\hat{\rho}(\theta) = e^{-i\hat{H}\theta}\hat{\rho}e^{i\hat{H}\theta}$$

$$P(x|\theta) = Tr(\hat{\rho}(\theta)\hat{E}_x)$$

$$S(\theta) = \frac{d}{d\theta} D[P(\theta + \delta\theta), P(\theta)]$$



Distances and speed depend on the choice of the observable.
In same cases it is possible to maximize over all POVM so that the distances & speeds do not depend on the observable but only on the generator of the transformation.

Families of statistical distances and speeds

$$(d_\alpha(p,q))^\alpha = \frac{1}{2}\sum_x \left| p_x^{\frac{1}{\alpha}} - q_x^{\frac{1}{\alpha}} \right|^\alpha \longrightarrow$$

Generalized Fisher information

$$f_\alpha[p(\theta)] = \sum_x p_x(\theta) \left| \frac{\partial}{\partial\theta} \log p_x(\theta) \right|^\alpha$$

$\alpha = 1$

$$\frac{1}{2}\sum_x |p_x - q_x|$$

Trace distance

$\alpha = 2$

$$\frac{1}{2}\sum_x (\sqrt{p_x} - \sqrt{q_x})^2$$

Fidelity

Hierarchy

$$F_1[\rho(\theta)] \leq F_\alpha[\rho(\theta)]^{\frac{1}{\alpha}} \leq \sqrt{F_2[\rho(\theta)]}$$

$$(\mathrm{d}_\alpha(p,q))^\alpha = \frac{1}{2}\sum_x |p_x - q_x|^\alpha \longrightarrow$$

Schatten speed

$$\mathrm{f}_\alpha[p(\theta)] = \left(\sum_x \left| \frac{\partial}{\partial\theta} p_x(\theta) \right|^\alpha \right)^{\frac{1}{\alpha}}$$

$\alpha = 2$

$$\frac{1}{2}\sum_x |p_x - q_x|^2$$

Hilbert-Schmidt

Hierarchy

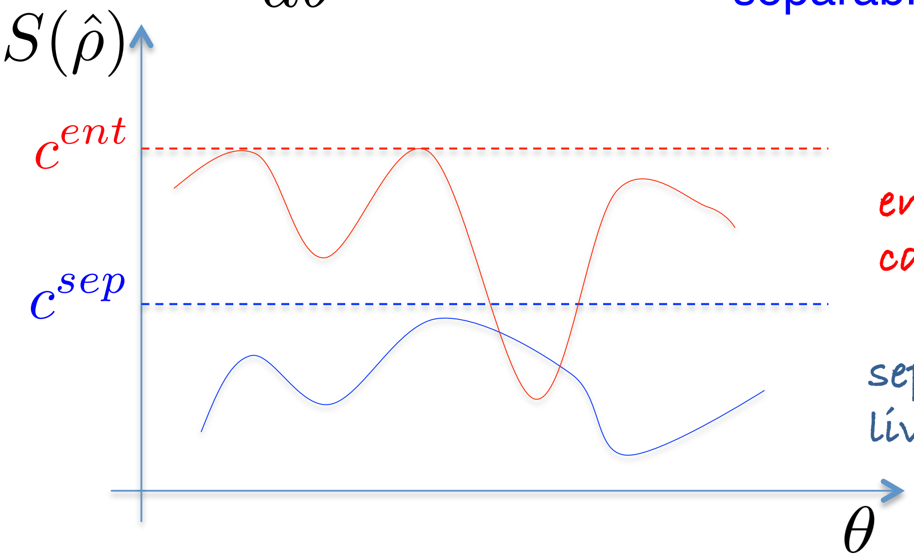
$$\mathrm{F}_\infty[\rho(\theta)] \leq \mathrm{F}_\alpha[\rho(\theta)] \leq \mathrm{F}_1[\rho(\theta)]$$

M. Gessner and A. Smerzi, PRA, 2018

In quantum mechanics,
all known statistical speeds have two speed limits:

$$S(\hat{\rho}) = \frac{d}{d\theta} D(\hat{\rho}(\theta), \hat{\rho}) \quad \leq c^{sep} \quad \leq c^{ent}$$

separable states — entangled states



E.g. : N qubits along a path generated by N local Hamiltonians $c^{sep} = N$, $c^{ent} = N^2$

L. Pezze` and A. Smerzi, PRL, 2009

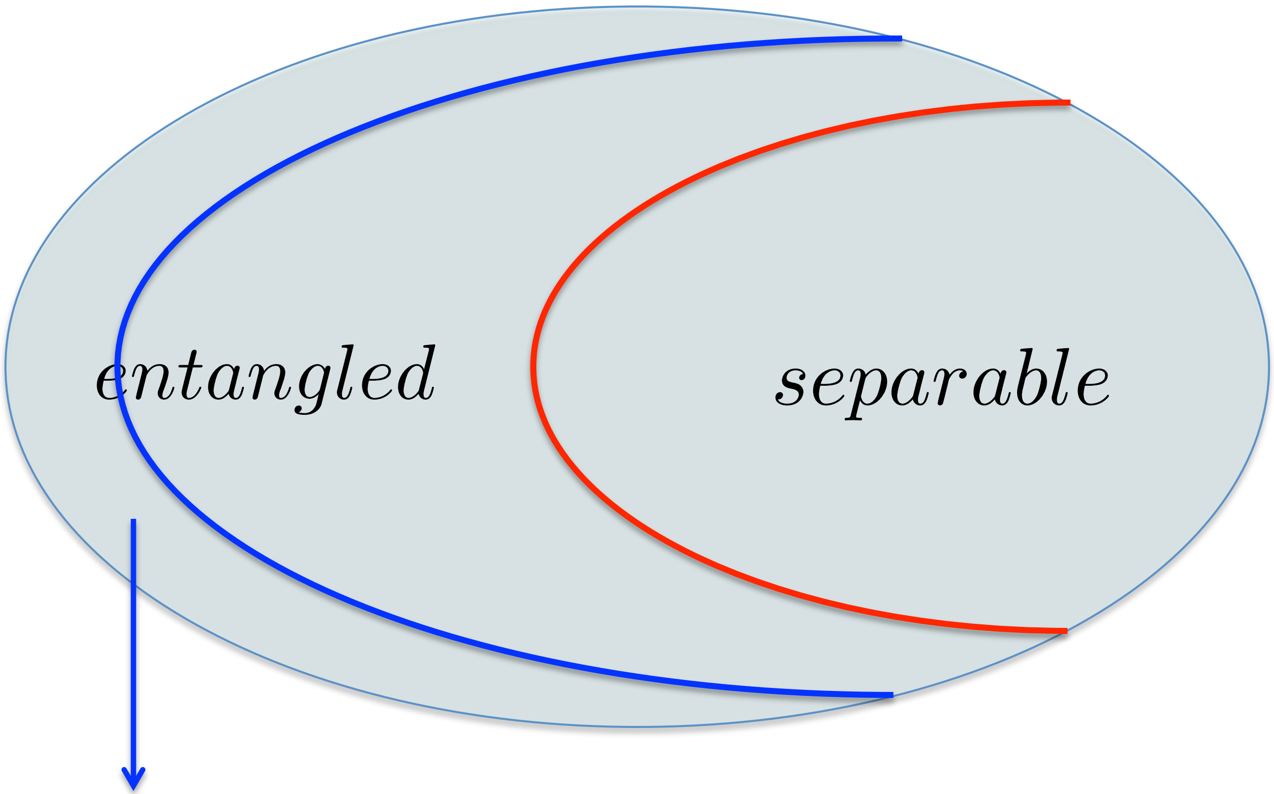
Why all separable states have a speed limit?

Don't know simple reason. Consequence of Hilbert space structure and unitary transformations

Why a speed limit for entangled states?

Perhaps no-signaling (special relativity)

Entangled states recognized by statistical speeds

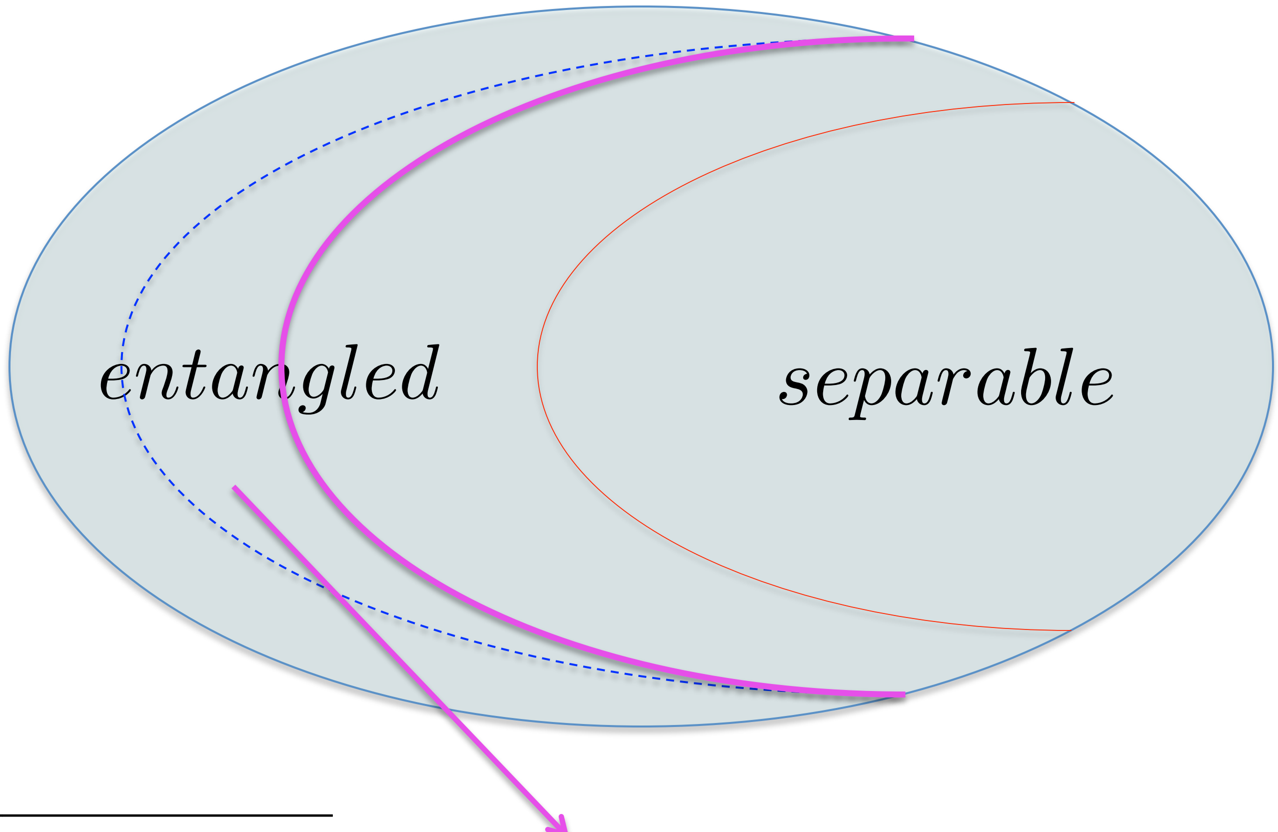


$$\sqrt{\sum_n \left(\frac{\partial P_n}{\partial \theta}\right)^2}$$

Hilbert-Schmidt

entangled if
$> c_{sep}$

Entangled states recognized by statistical speeds



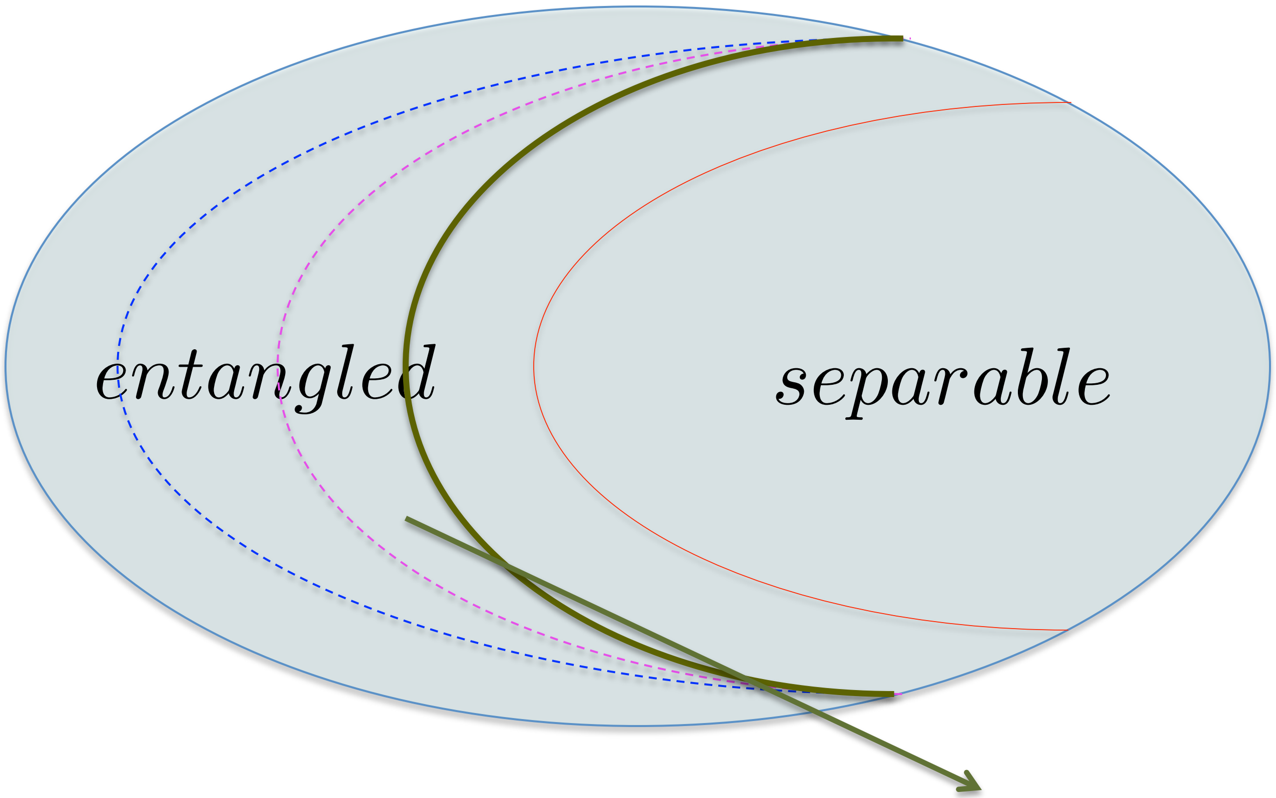
$$\sqrt{\sum_n \left(\frac{\partial P_n}{\partial \theta}\right)^2} \leq \sum_n \left|\frac{\partial P_n}{\partial \theta}\right| \leq$$

entangled if $> c_{sep}$

Hilbert-Schmidt

Kolmogorov

Entangled states recognized by statistical speeds

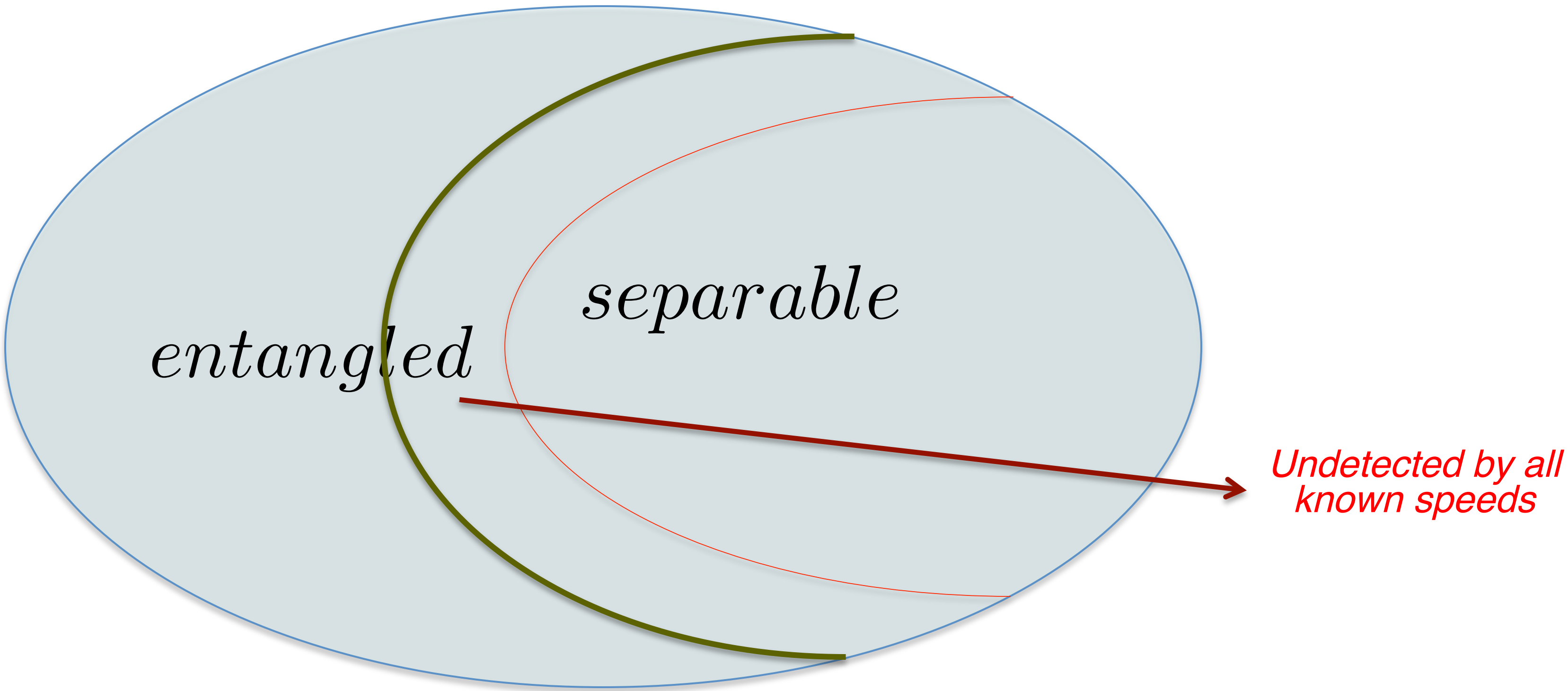


$$\sqrt{\sum_n \left(\frac{\partial P_n}{\partial \theta}\right)^2} \leq \sum_n \left|\frac{\partial P_n}{\partial \theta}\right| \leq \sqrt{\sum_n \frac{1}{P_n}\left(\frac{\partial P_n}{\partial \theta}\right)^2}$$

entangled if $> c_{sep}$

Hilbert-Schmidt Kolmogorov Fisher information

Some entangled are not recognized by all known statistical speeds

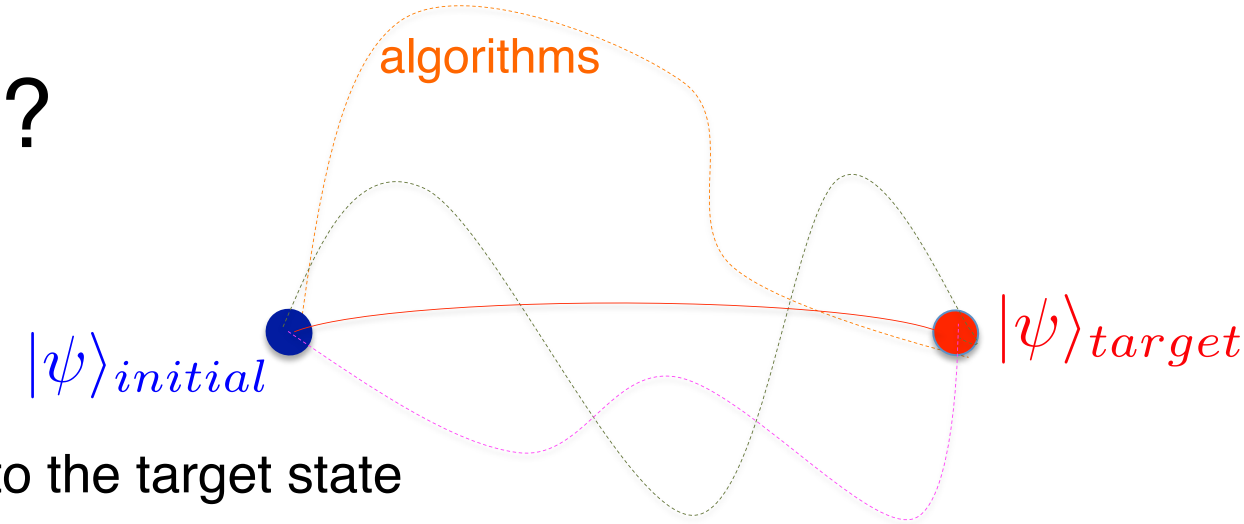


$$\sqrt{\sum_n \left(\frac{\partial P_n}{\partial \theta}\right)^2} \leq \sum_n \left|\frac{\partial P_n}{\partial \theta}\right| \leq \sqrt{\sum_n \frac{1}{P_n}\left(\frac{\partial P_n}{\partial \theta}\right)^2}$$

entangled if $> c_{sep}$

Hilbert-Schmidt   Kolmogorov   Fisher information

What is a QT?



Typical QT problem:
go from the initial state to the target state
as fast as possible

$$|\langle\psi_{initial}|\psi_{target}\rangle| \sim 0$$

Simplest QT problem: Minimize the time needed by two probability distributions to become distinguishable.

$$D = \int_{\theta_{initial}}^{\theta_{target}} S(\theta)\ d\theta \sim 1$$

You go faster if, and only if, the state contains entanglement **recognized** by the statistical speed

$$S(\theta) \quad \leq c^{sep} \qquad \leq c^{ent}$$

separable states   entangled states

Different quantum technologies require different classes of entangled states. In some known cases, the different classes are provided by different statistical speeds. *Maybe* this is general.

Example #1: The class of entangled states detected by the Fisher info provides sub shot-noise phase estimation sensitivity

Example #2: The class of entangled states recognized by the Kolmogorov speed useful to distinguish two distributions with a single yes/no measurement

Kolmogorov distance: $$d(p,q) = \frac{1}{2}\sum_x |p_x - q_x|.$$

Probability to successfully identify one of two distributions from a **single** yes/no measurement.

Example #3: The class of entangled states recognized by the Fubini-Study metric useful for Grover search quantum algorithm

Example #4: Local-realism violated by the class of entangled states recognized by the Hilbert-Schmidt speed