Measuring the impact of changing the formulation of a mathematical task

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MESE 1

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Starting point:

When facing a mathematical task, students are influenced by the formulation of the task itself.

Theoretical issues

Operational issues

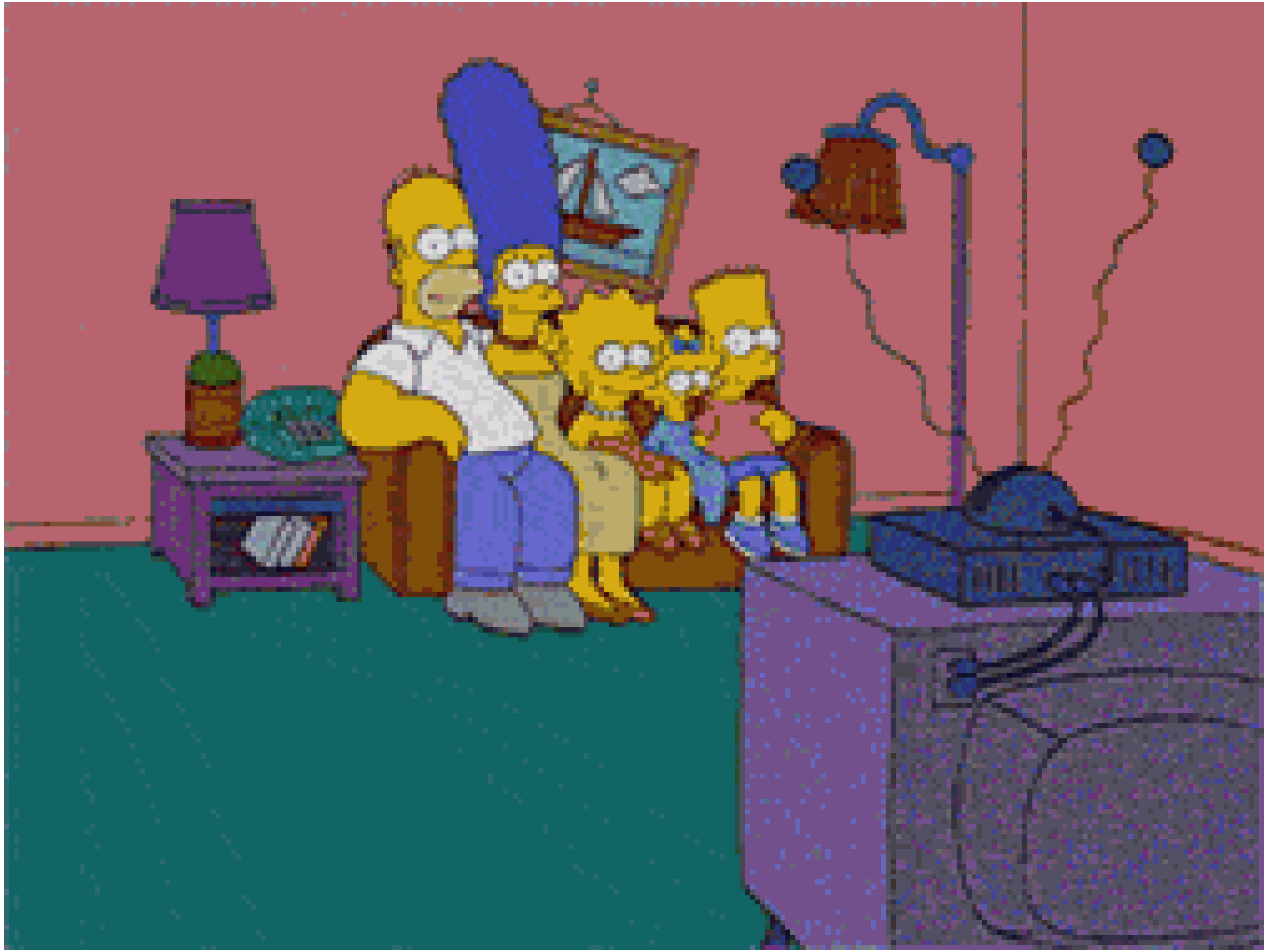
Didactic issues

Nesher 1982, Duval 1991, Laborde 1995, Thevenot & Oakhill, 2005, De Corte et al 1988, Daroczy et al. 2015, Franck et al. 2007, Branchetti and Viale 2015

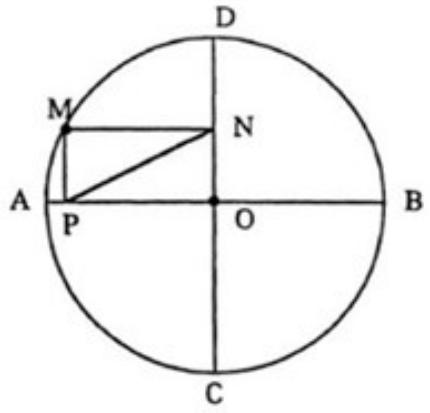
A problematic task for the researcher:

How to measure this impact?

How to rule the chaos?



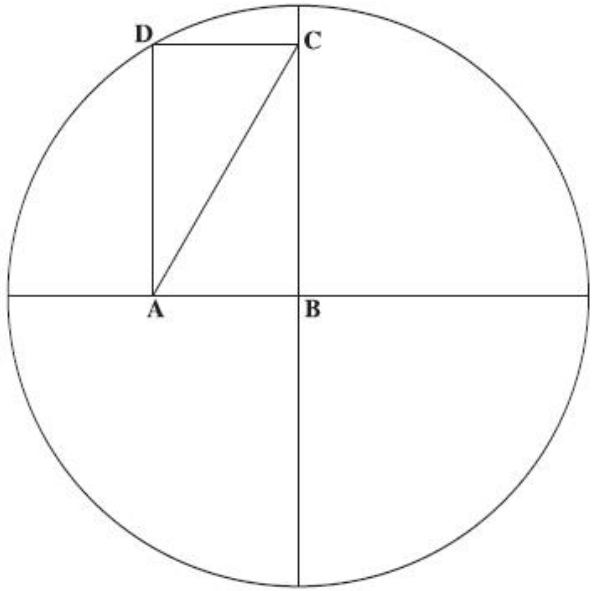
Efraim Fischbein (1993):
The Theory of Figural Concepts



In a circle with its center in O we draw two perpendicular diameters AB and CD.
We choose arbitrarily a point M and we draw the perpendiculars MN and MP on the two diameters. What is the length of PN?"

*The fact that we jump to the conclusion **suddenly** - PN=MO=radius=constant - at the very moment when we have grasped the rectangle PONM, without an intervening investigation, supports the idea that the considered figure is, from the beginning, not an ordinary image but an already logically controlled structure*

D23. **La circonferenza in figura ha il raggio di 4 cm. ABCD è un rettangolo.**



a. **Qual è la lunghezza (in cm) del segmento $\overline{AC}$?**

Risposta: .....

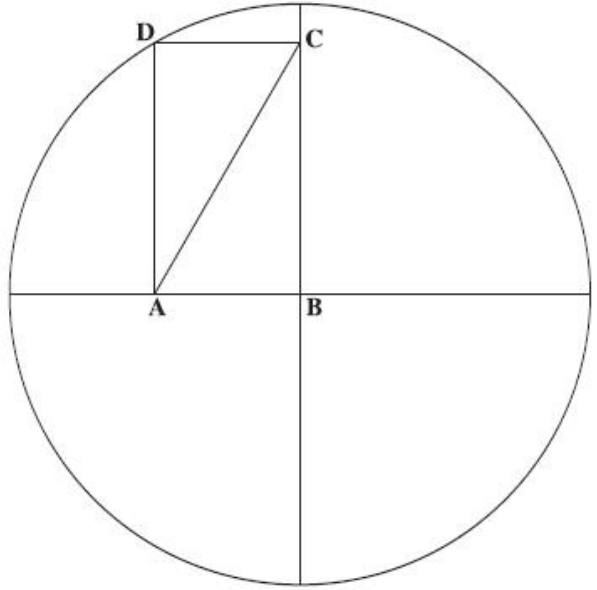
b. **Giustifica la tua risposta:**

.....

.....

.....

D23. **The circle represented has a radius of 4 cm. ABCD is a rectangle.**



a. **What is the length (in cm) of the segment $\overline{AC}$?**

Answer : .....

b. **Explain your reasoning**

.....

.....

.....

Invalsi, Grade 8, 2010, National assessment system

Syntax deals with the ways in which words can combine, with the structures (or constructions) that are thus obtained and with the effects that such combinations have on other planes, such as that of pragmatics. In fact, syntax has contacts with various levels of the language [...] syntactic structures have pragmatic effects

RQ1. Is there a significant quantitative difference between the distributions of students' answers if the syntactic formulation of the task changes? Are there syntactic formulations that correspond to a higher percentage of choices of the option including the theoretical property of the rectangle?

RQ2. Is it possible to identify and qualitatively interpret the students' reasoning supporting their choice, linking the elements of the syntactic organization of the text to the argumentative processes and to the interaction text-figure? Is there evidence of an "implication" between features of the task formulation and students' recurrent reasonings?

Theoretical framework

Boero et al 2010; Chazan 1993; Durand-Guerrier et al 2011; Toulmin 2003

Features of the formulation of a mathematical task

Conception of geometric figures

Toulmin's model for argumentation

Ferrari 2004; Daroczy et al. 2015;

Laborde 2005; Herbst et al 2017; Fischbein 1993; Fujita et 2013; Larkin & Simon 1997

Research Design

(weak) multilevel sequential explanatory

Step 0
LSA task derived from Fischbein's
Stratified sample of 25,626 students
Analysis of LSA results
CTT and IRT

Step 1
4 formulations of the task
Categorization via Toulmin model
Stratified sample of 1684 students
Anchoring technique

Step 2
2 out of 4 of the previous formulations
Meta-reflection questionnaire
24 students of the same class
Semi-structured paired interviews
14 students of the class

Creswell & Plano-Clark 2017

Step 0: Discussing empirical evidence from LSA

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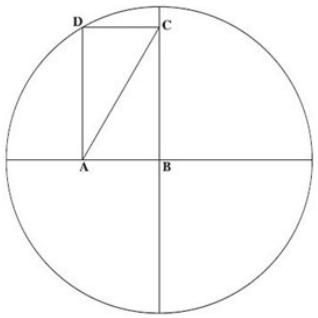

a. **Qual è la lunghezza (in cm) del segmento $\overline{AC}$?**

Risposta: .....

b. **Giustifica la tua risposta:**

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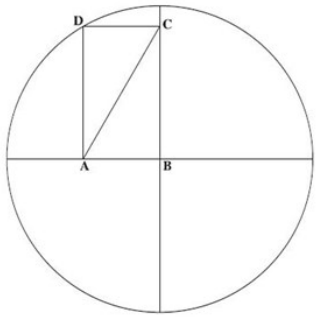

a. **What is the length (in cm) of the segment $\overline{AC}$?**

Answer : .....

b. **Explain your reasoning**

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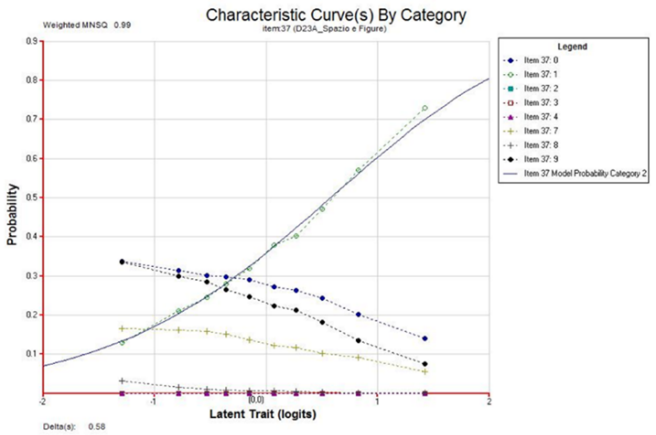

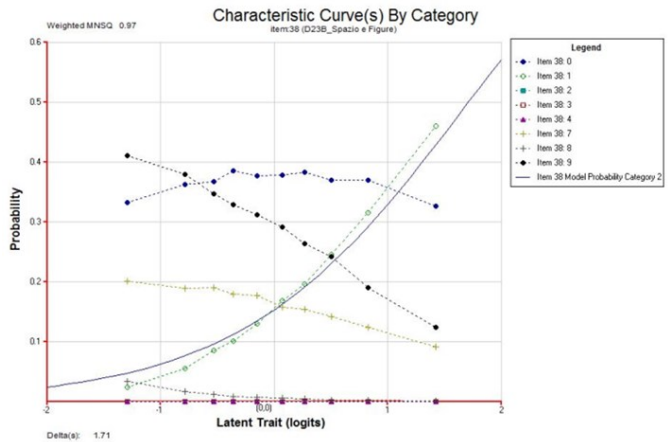


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item:37 (D23A_Spazio e Figure)
Cases for this item  25626   Discrimination  0.38
Item Threshold(s):     0.58   Weighted MNSQ   0.99
Item Delta(s):      0.58
-------------------------------------------------------------------------------
 Label    Score     Count   % of tot  Pt Bis     t  (p)   PV1Avg:1 PV1 SD:1
-------------------------------------------------------------------------------
   0       0.00     6818      26.61   -0.13   -21.37(.000) -0.16     0.71
   1       1.00     9570      37.34    0.38    66.68(.000)  0.35     0.76
   7       0.00     3231      12.61   -0.11   -17.75(.000) -0.21     0.69
   8       0.00      222       0.87   -0.10   -15.85(.000) -0.77     0.76
   9       0.00     5785      22.57   -0.20   -31.94(.000) -0.26     0.68
===============================================================================
item:38 (D23B_Spazio e Figure)
Cases for this item  25626   Discrimination  0.36
Item Threshold(s):     1.71   Weighted MNSQ   0.97
Item Delta(s):      1.71
-------------------------------------------------------------------------------
 Label    Score     Count   % of tot  Pt Bis     t  (p)   PV1Avg:1 PV1 SD:1
-------------------------------------------------------------------------------
   0       0.00     9345      36.47   -0.00    -0.49(.621) -0.01     0.74
   1       1.00     4555      17.77    0.36    61.32(.000)  0.55     0.73
   7       0.00     4102      16.01   -0.09   -15.04(.000) -0.16     0.71
   8       0.00      227       0.89   -0.10   -16.02(.000) -0.78     0.75
   9       0.00     7397      28.87   -0.20   -33.06(.000) -0.22     0.70
===============================================================================
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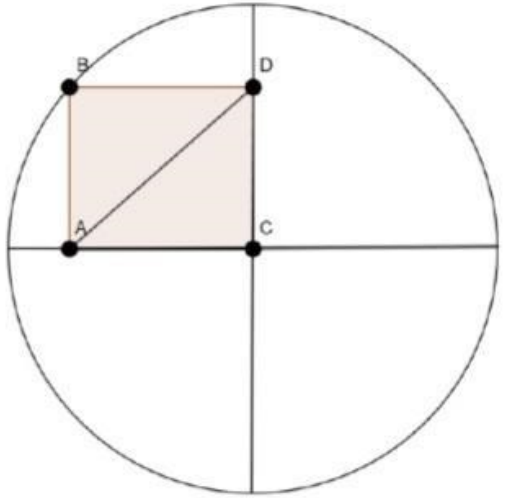
Step 1: Implementing variations in the syntactic formulation of the task: the stimulus

Version 1 *D1. D2. C.*	*Version 2* *D1 → D2. C.*	*Version 3* *D1 ↓ C ↓ D2.*	*Version 4* *C ↓ (D1, D2).*
C is a circle with centre C and radius 4 cm. CABD is a rectangle. Mario states that the segment AD measures 4 cm.	**In the circle C with centre C and radius 4 cm, CABD is a rectangle. Mario states that the segment AD measures 4 cm.**	**Knowing that the circle C has centre C and radius 4 cm, Mario states that the segment AD measures 4 cm, taking into account that CABD is a rectangle.**	**Mario states that the segment AD measures 4 cm, since the circle C has centre C and radius 4 cm and CABD is a rectangle.**

- C the claim "Mario states that the segment AD measures 4 cm."
- D1 the facts (ground) related to the circle "C is a circle with centre C and radius 4 cm."
- D2 the fact (ground) related to the rectangle "CABD is a rectangle"
- W1 the warrant "The length of AB can be calculated using the Pythagorean Theorem and the result is different from 4"
- W2 the warrant "There are not enough data for calculating the length of AD"
- W3 the warrant "The length of AB can be calculated using the Pythagorean Theorem and the result is 4"
- W4 the warrant "The diagonals of a rectangle are equal"

Step 1: Implementing variations in the syntactic formulation of the task: the question

D10. Look at the picture.



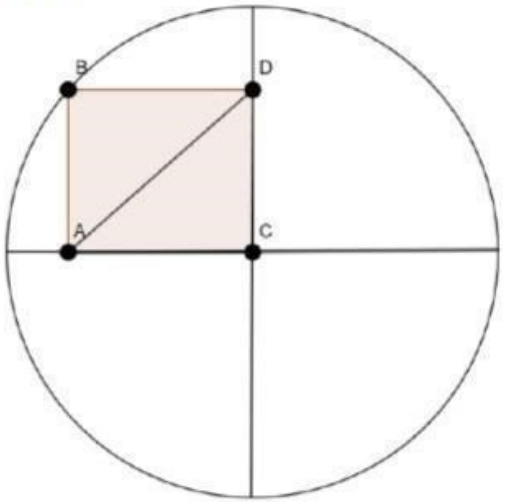
Version 1 *D1. D2. C.*	*Version 2* *D1 → D2. C.*	*Version 3* *D1 ↓ C ↓ D2.*	*Version 4* *C ↓ (D1, D2).*
C is a circle with centre C and radius 4 cm. CABD is a rectangle. Mario states that the segment AD measures 4 cm.	**In the circle C with centre C and radius 4 cm, CABD is a rectangle. Mario states that the segment AD measures 4 cm.**	**Knowing that the circle C has centre C and radius 4 cm, Mario states that the segment AD measures 4 cm, taking into account that CABD is a rectangle.**	**Mario states that the segment AD measures 4 cm, since the circle C has centre C and radius 4 cm and CABD is a rectangle.**

Is Mario right? Choose an answer and a motivation.

Mario is NOT right because		Mario is right because	
The length of AD can be calculated using the Pythagorean Theorem and the result is different from 4	**There are not enough data for calculating the length of AD**	**The length of AD can be calculated using the Pythagorean Theorem and the result is 4**	**The diagonals of a rectangle are equal**
A	**B**	**C**	**D**

Step 1: Implementing variations in the syntactic formulation of the task: the question

D10. Osserva la figura



Version 1 *D1. D2. C.*	*Version 2* *D1 → D2. C.*	*Version 3* *D1 ↓ C ↓ D2.*	*Version 4* *C ↓ (D1, D2).*
La circonferenza rappresentata in figura ha centro C e raggio 4 cm. CABD è un rettangolo. Mario afferma che il segmento AD misura 4 cm.	Nella circonferenza rappresentata in figura di centro C e raggio 4 cm. CABD è un rettangolo. Mario afferma che il segmento AD misura 4 cm.	Sapendo che la circonferenza rappresentata in figura ha centro C e raggio 4 cm, Mario afferma che il segmento AD misura 4 cm, tenendo conto che CABD è un rettangolo.	Mario afferma che il segmento AD misura 4 cm, visto che la circonferenza rappresentata in figura ha centro C e raggio 4 cm e CABD è un rettangolo.

Mario ha ragione? Scegli una risposta con il suo perchè.

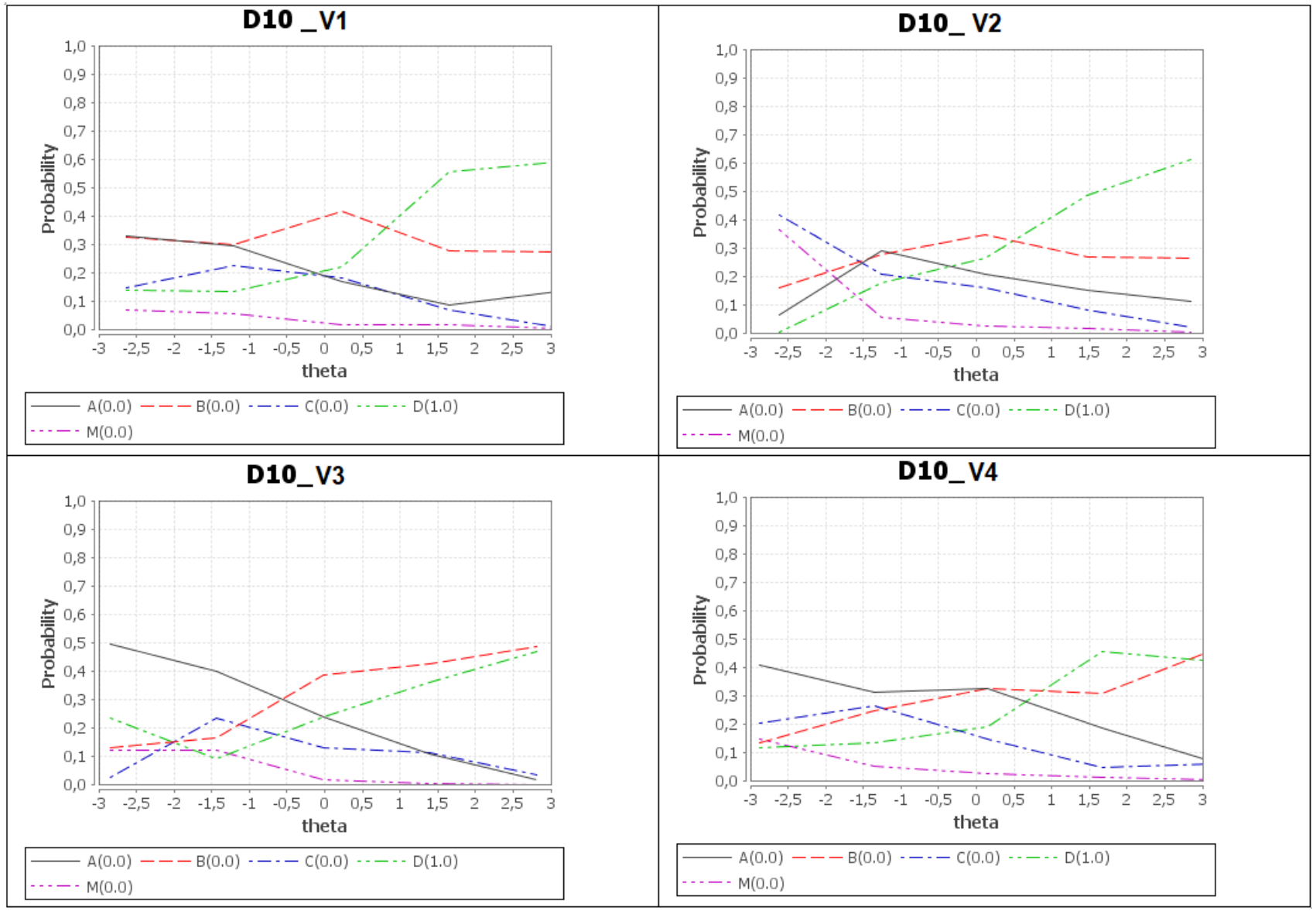
Mario NON ha ragione perchè		Mario HA ragione perchè	
La lunghezza di AD si può calcolare usando il teorema di Pitagora e il risultato è diverso da 4	Non ci sono abbastanza dati per calcolare la lunghezza di AD	La lunghezza di AD si può calcolare usando il teorema di Pitagora e il risultato è 4	Le diagonali di un rettangolo sono congruenti tra di loro
A	B	C	D

Step 1: Implementing variations in the syntactic formulation of the task: results

Version 1 *D1. D2. C.*	*Version 2* *D1 → D2. C.*	*Version 3* *D1 ↓ C ↓ D2.*	*Version 4* *C ↓ (D1, D2).*
C is a circle with centre C and radius 4 cm. CABD is a rectangle. Mario states that the segment AD measures 4 cm.	**In the circle C with centre C and radius 4 cm, CABD is a rectangle. Mario states that the segment AD measures 4 cm.**	**Knowing that the circle C has centre C and radius 4 cm, Mario states that the segment AD measures 4 cm, taking into account that CABD is a rectangle.**	**Mario states that the segment AD measures 4 cm, since the circle C has centre C and radius 4 cm and CABD is a rectangle.**

Item	Difficulty	Std.Err
V 1	0.97	0.12
V 2	0.86	0.12
V 3	1.14	0.13
V 4	1.37	0.13

Step 1: Implementing variations in the syntactic formulation of the task



Step 1: Implementing variations in the syntactic formulation of the task

The difficulty parameter increases with the syntactic complexity

In V3 and V4 the increased linguistic complexity seems to lead more students to a wrong choice. A larger number of students chose distractor A, hence warrant W1.

V1 discriminates better than the other versions

Step 2: Qualitative investigation on students' reasoning and search for recurrent approaches

Reflection Questionnaire

"Existence proofs", Schoenfeld 2000

Paired interviews

Step 2: Qualitative investigation on students' reasoning and search for recurrent approaches: aspects evidenced

1. linguistic issues;
2. sentential/graphical interaction and different students' approaches while managing sentential and graphical information in problem solving;
3. spatiographical-theoretical elaboration of information presented by means of diagrams;
4. conceptions of figures and their impact on students' interpretation of the task and use of text and diagrams and set of controls.

Step 2: Qualitative investigation on students' reasoning and search for recurrent approaches

	actions on the figure	use of information in the text	argumentative reasoning	use of sentential statements (data program)
graphical/ figural	actions on the figure to respond on a perceptual basis, without elaboration	comparison between data in the text and characteristics of the figure	arguments starting from observations on the figure	sentential sentences relating to observed / graphic properties
theoretical/logical	actions on the figure to logically argue / apply properties obtained on a figural basis but used in a conceptual way	data / information from the text relating to conceptual properties and / or to argue	arguments in theoretical / logical form	use of sentential sentences relating to theoretical/ conceptual properties
numerical data/measurement	actions on the figure related to the measurements	data information from text to calculate / approximate (R: calculate)	arguments relating to measurements /values/ results of operations	sentential statements relating to the applicability of formulas

Step 2: Qualitative investigation on students' reasoning and search for recurrent approaches: categories of approaches

1. Students who chose the option with a conceptual warrant (D) following an argumentative logically controlled structure of reasoning, who carried out reasoning mainly based on the figure
2. Student who uses the figure exploiting mainly the figural/spatio-graphical aspects (figure as a diagram), not considering theoretical geometrical properties and not argumenting
3. Students who chose an option mentioning calculation and Pythagoras theorem and followed a procedural/computational approach consistent with the conception of figures as descriptions in the microspace (measurements, formulas, calculation)
4. Students focusing only on data in the text, who chose the option "Not enough data"

Conclusion

Answer to Q1

The answer to research question 1 is affirmative: we observed significant differences between the distributions of students' choice of the four options in case of different syntactic formulations of the task. The trend goes in the following direction: increasing the linguistic complexity →increasing the difficulty of the task (i.e., reduction of correct answers). Syntactic complexity makes an already intrinsically difficult job even harder.

Conclusion

Answer to Q2

Our analysis showed that the students' reasonings and strategies cannot be easily and linearly connected to the formulation.

The syntactic formulation of the text, thus the relationships established between different elements in the text, seems to have an impact on students' approaches, combined with other conditions.

Further issues

These qualitative results are an interesting starting point for further investigations about the complex phenomena that underlie geometrical problem solving with word problems and figures in the microspace at the transition between middle and high school. It is possible that eye-tracker analysis of the student's reading phase could help to integrate such data by looking at the starting point of a student's reading and to see whether and how the key elements of the text and of the figure are looked at, or not.

MESE 1.

GRAZIE!