Latent Stochastic Differential Equations for Modeling Quasar Variability and Inferring Black Hole Properties

The Restless Nature of AGN: 10 years later

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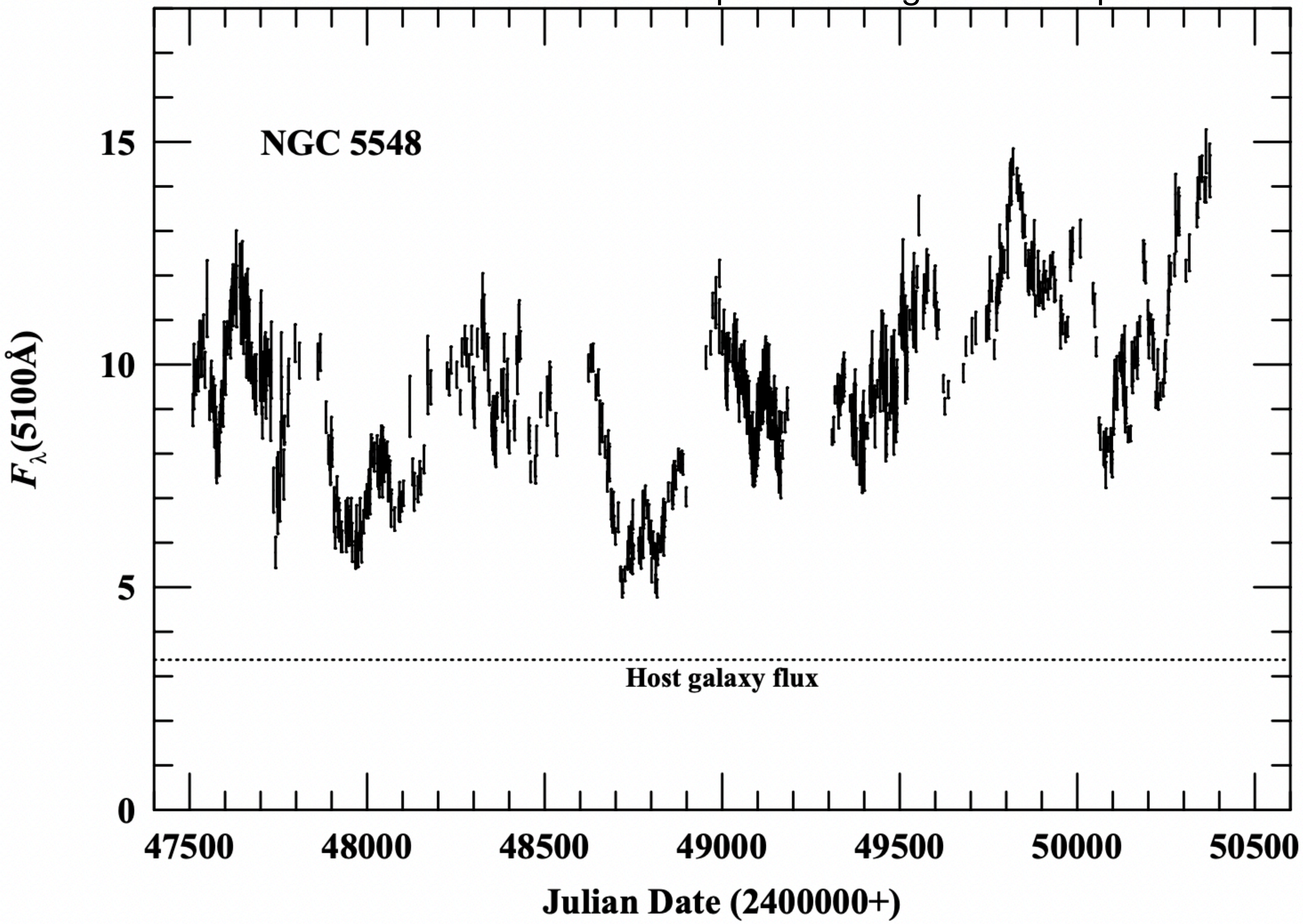
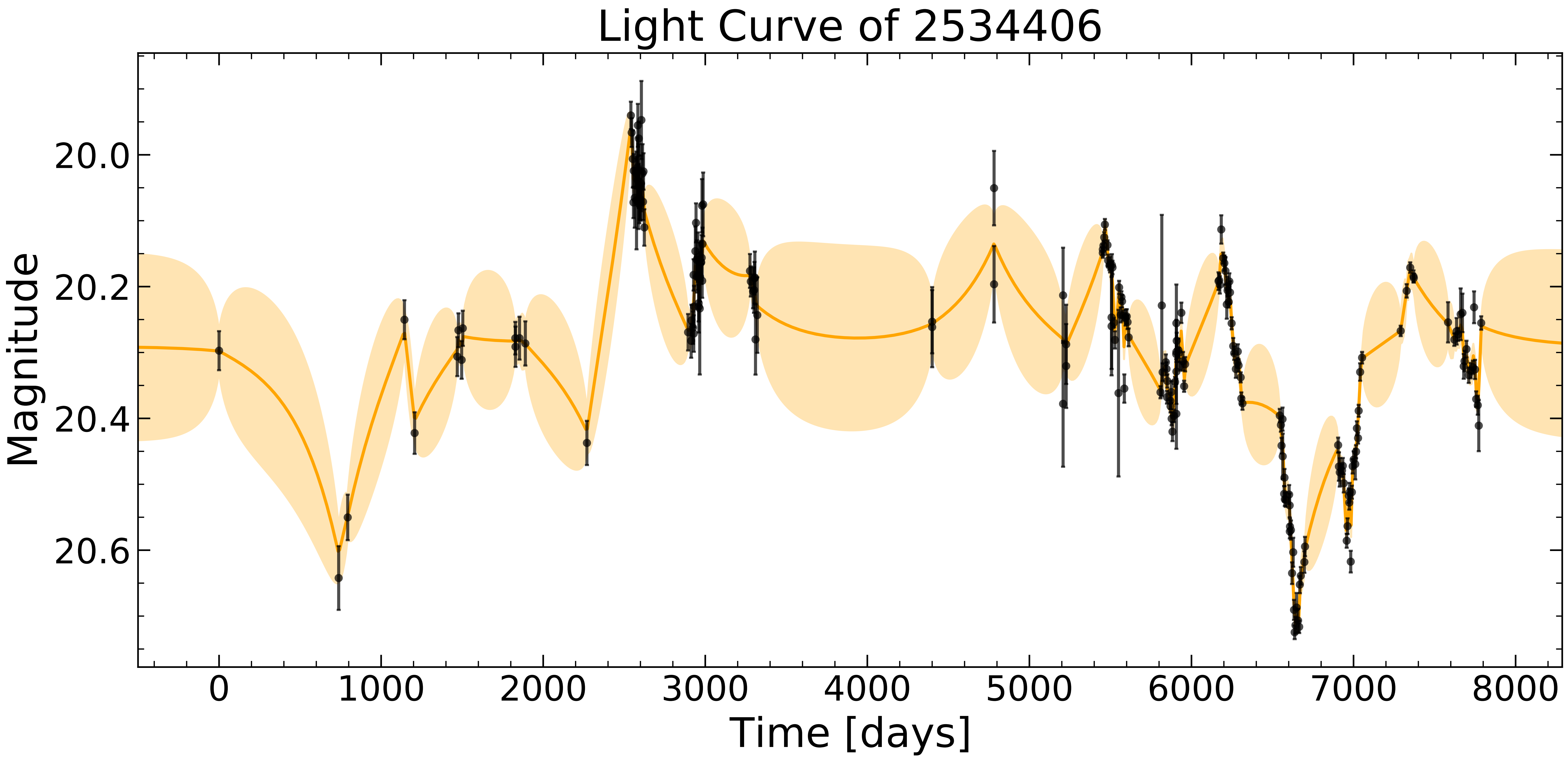
Optical Quasar Light Curves

Source: Peterson 2001 https://arxiv.org/abs/astro-ph/0109495



- LSST is set to observe tens of millions of quasar light curves in 6 optical bands but with long season gaps and sparsely sampled data
- We simulate LSST 10 year light curves and train a neural network to model the variability and infer black hole properties

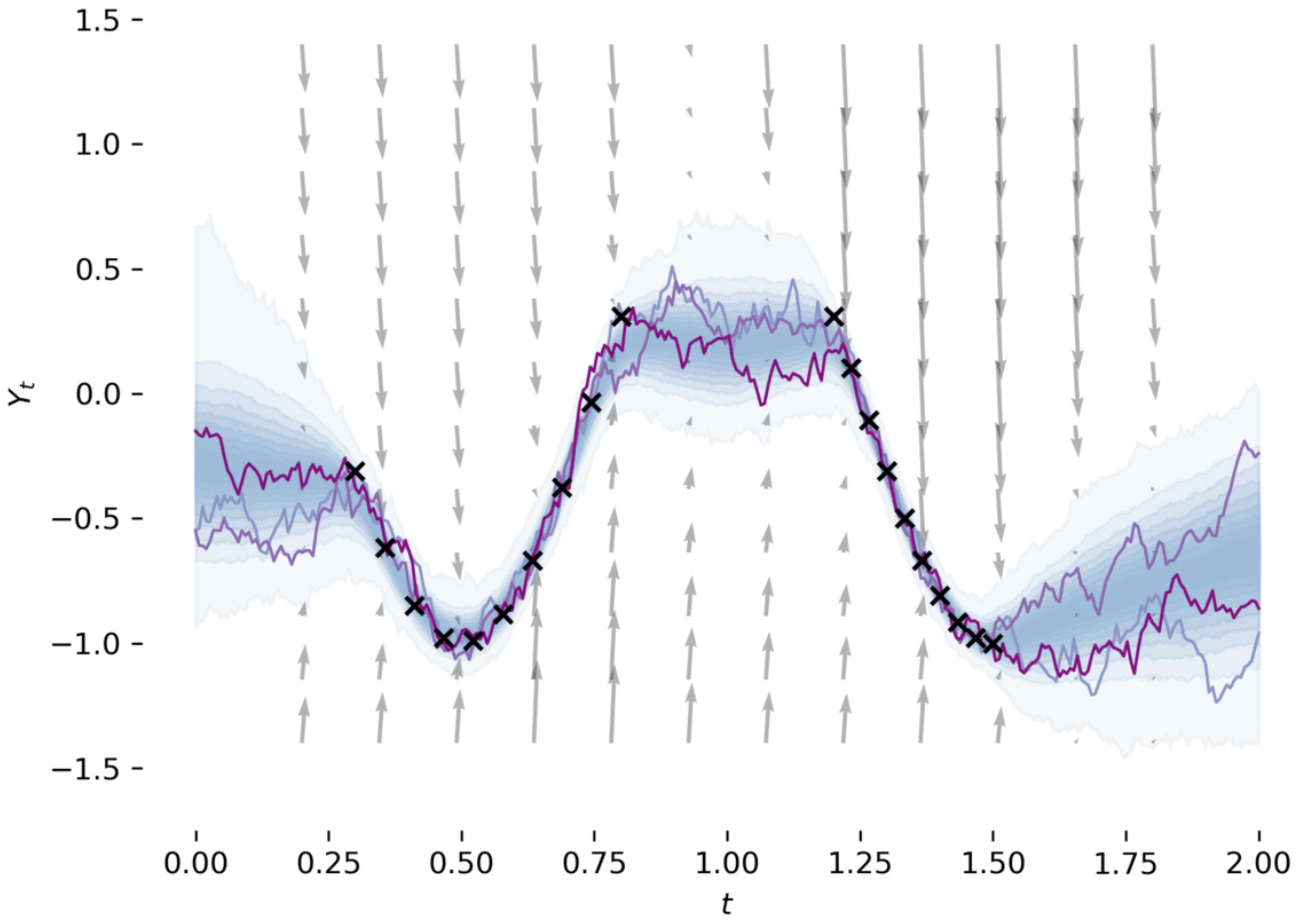
Gaussian Process Regression



Source: Stone et al. 2022 https://arxiv.org/abs/2201.02762

- Gaussian process regression (GPR) requires choosing a specific form of the kernel
- GPR can measure variability parameters from its kernel
- Using multi-band data is difficult
- Hard to apply to tens of millions of light curves

Latent Stochastic Differential Equations (SDEs)

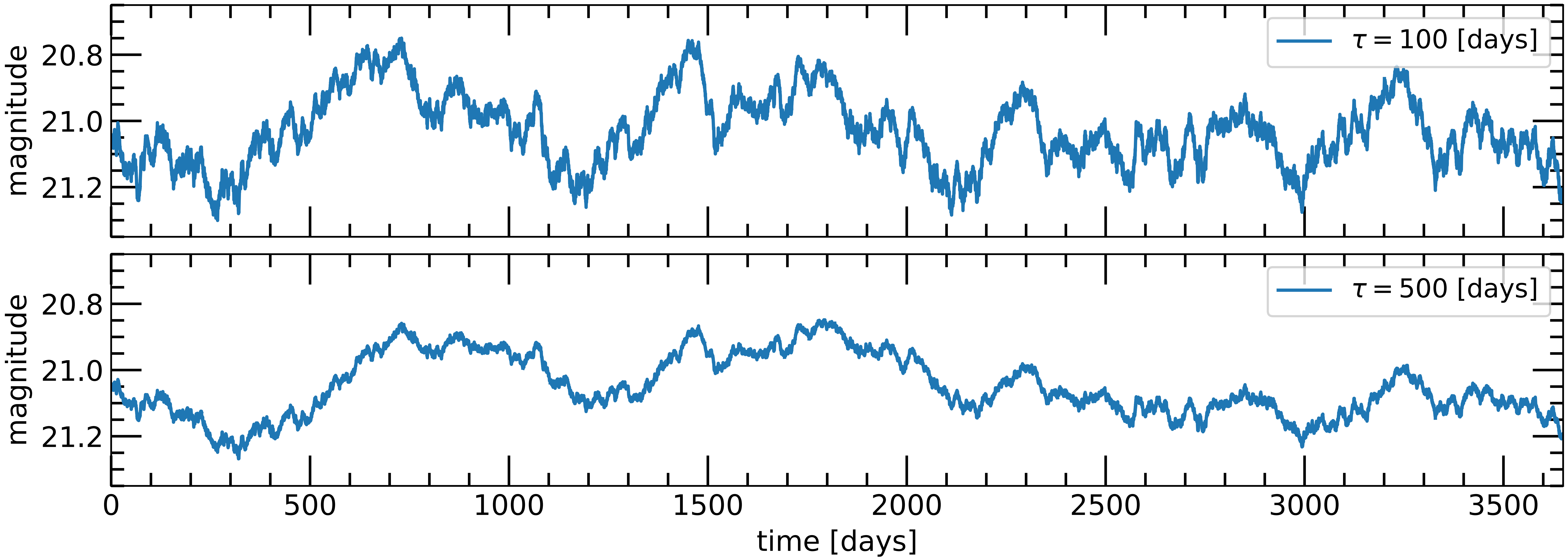


Overall goals:

- Use latent SDEs to fit quasar light curves (multivariate time series with missing data)
- Simultaneously infer black hole properties (parameter inference)

Source: https://github.com/google-research/torchsde (Li et al., 2020)

Driving Variability Model



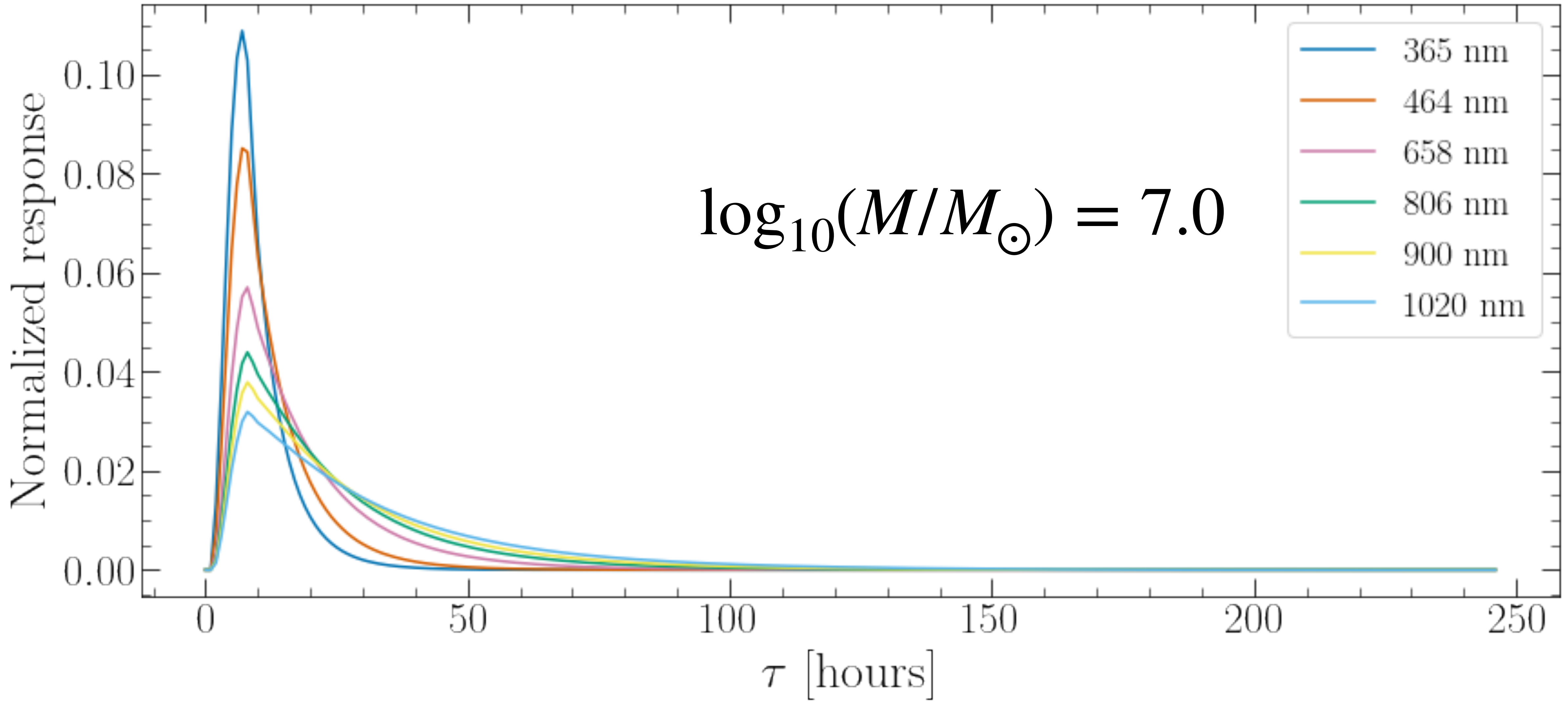
Damped random walk (DRW)

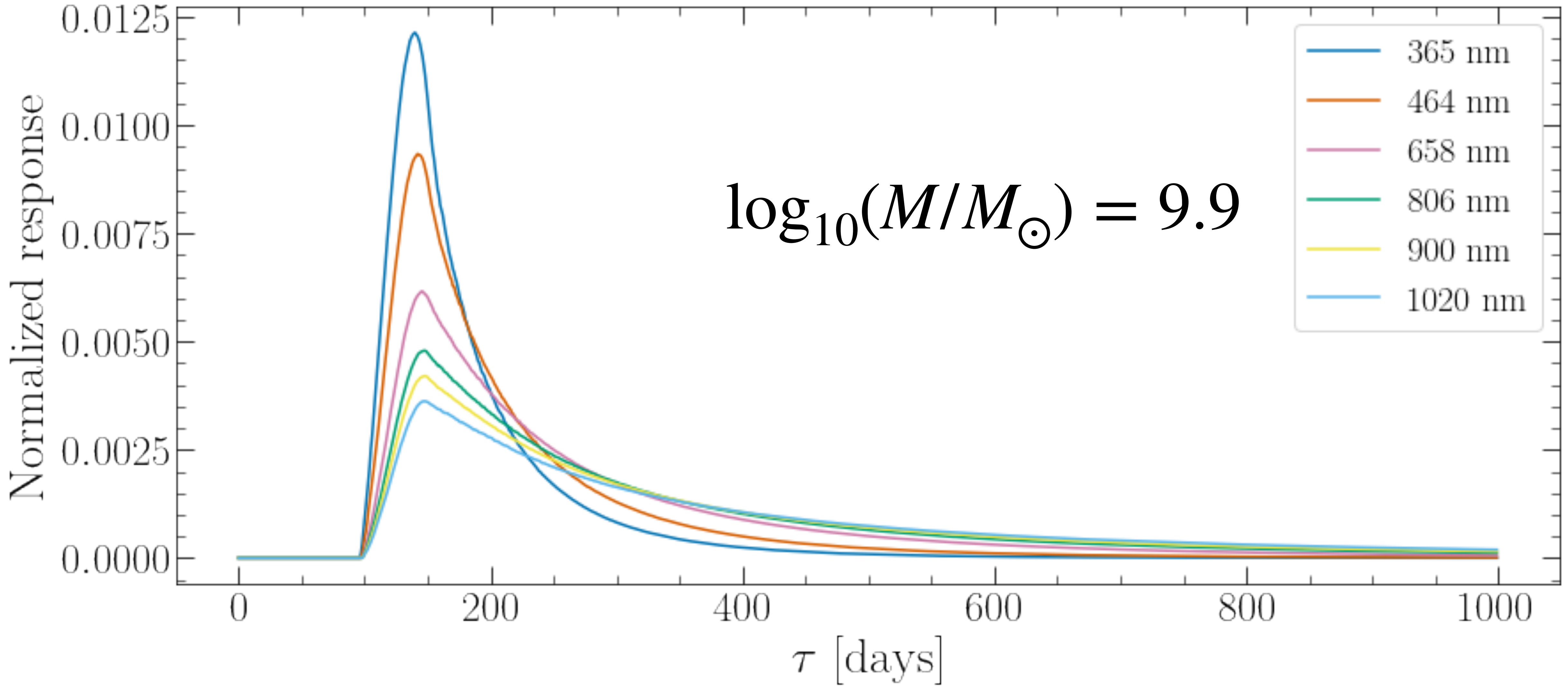
$$\mathrm{d}X(t) = -\frac{1}{\tau}X(t)\ \mathrm{d}t + \sigma\sqrt{\mathrm{d}t}\ \epsilon(t) + b\ \mathrm{d}t$$

- $\bar{X} = b\tau$ is the mean
- τ is characteristic time scale
- $\mathrm{SF}_\infty = \sigma\sqrt{\tau/2}$ standard deviation of fluctuation

Transfer Function

Source: Henry Best

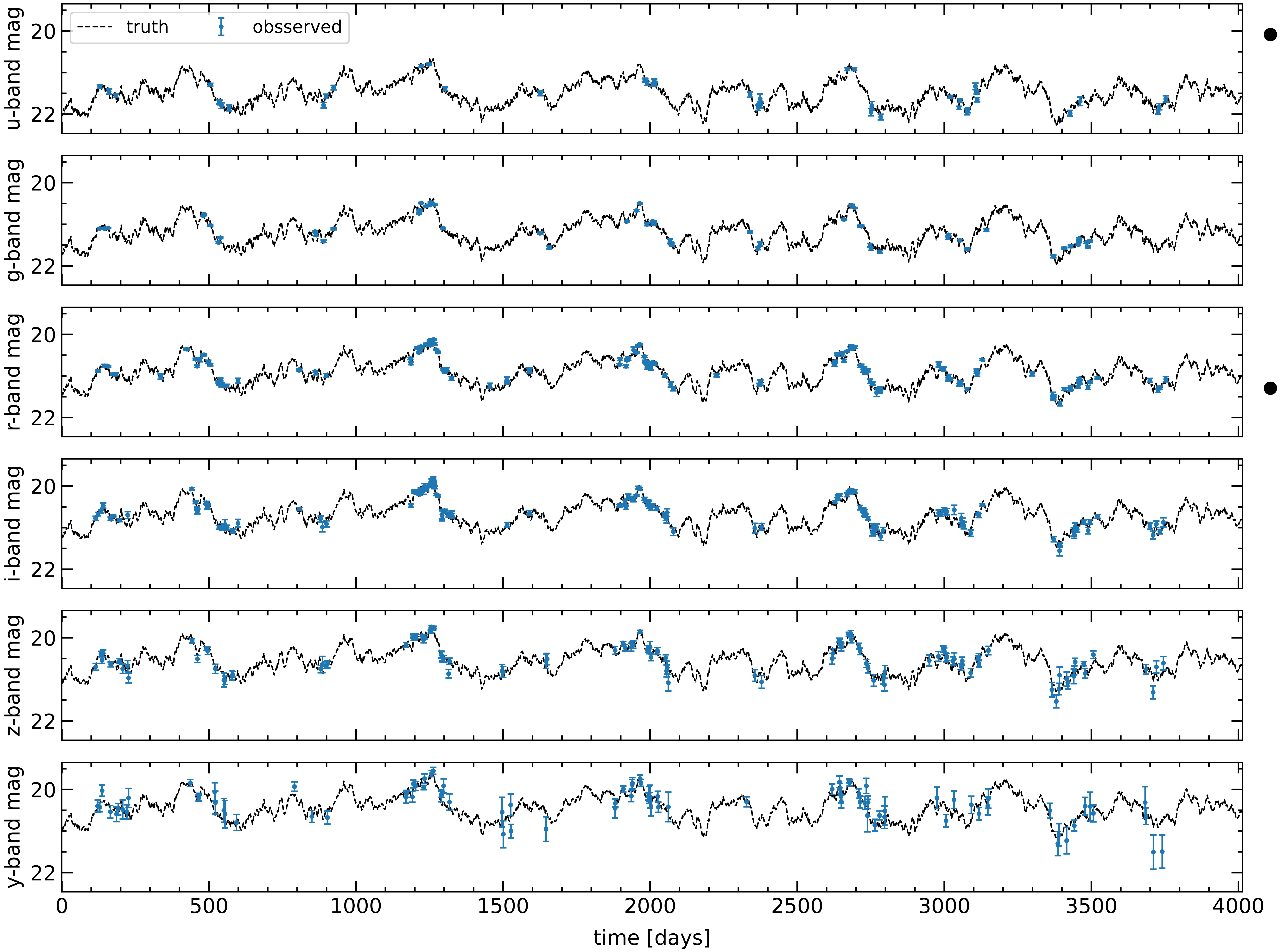




Name	Description	Min.	Max.	Unit
$\log_{10}(M/M_\odot)$	BH mass	6	10	-
a	BH spin	-1	1	-
θ_{inc}	inclination angle	1	80	deg
h	corona height	1	50	r_g
β	temp. power law	0.3	1.0	-
z	redshift	0.1	6	-
SF_∞	DRW amplitude	0	0.5	mag
$\log_{10}(\tau/\mathrm{day})$	DRW time scale	1	3.5	-

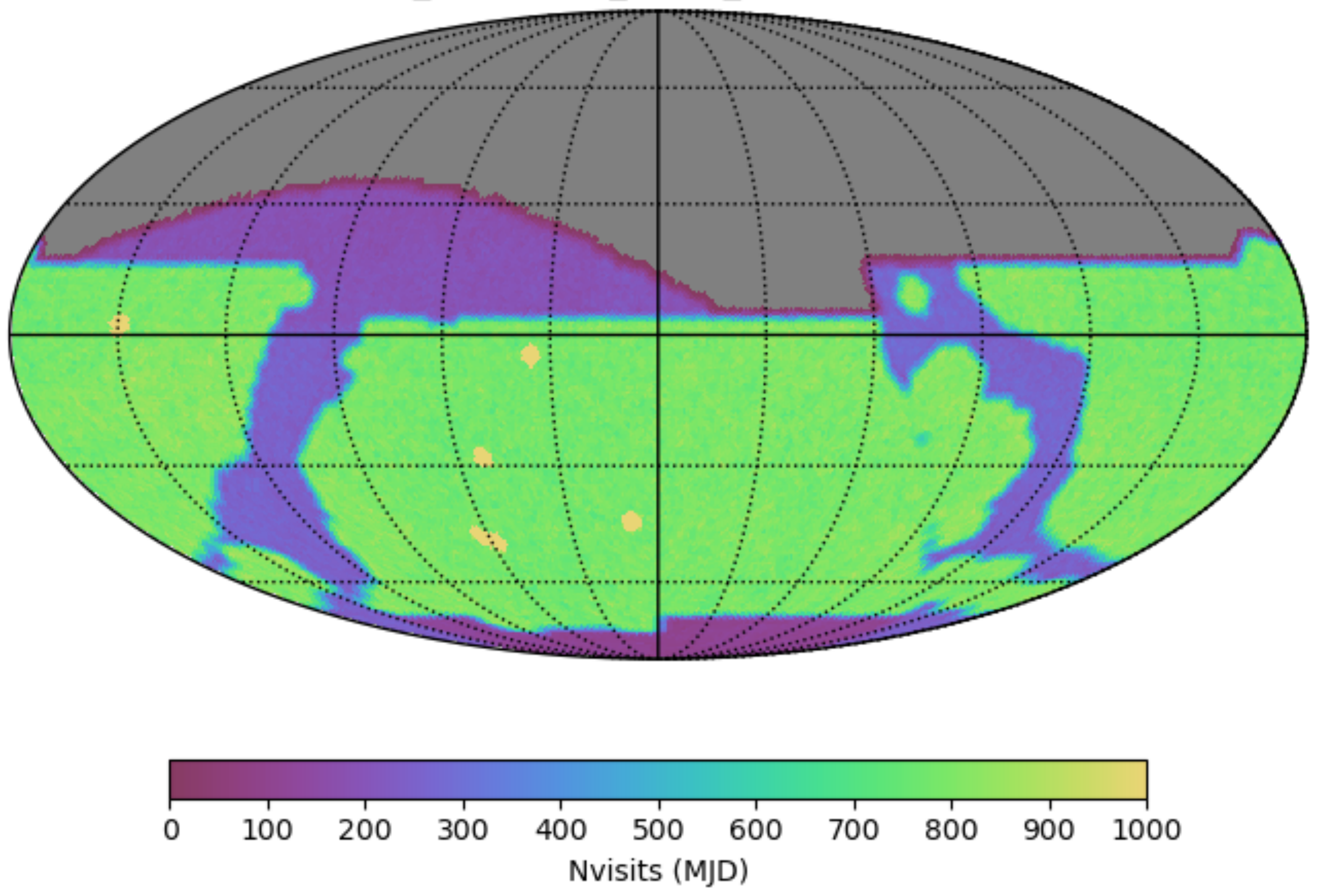
- Convolve driving variability with transfer function to get optical variability
- Modified thin-disk + lamp-post geometry
- Time delays between bands relate to black hole properties like the black hole mass

LSST Cadence and Noise



- We use rubin_sim (https://github.com/lsst/rubin_sim) to generate 100,000 mock LSST observations strategies from across the sky

- Combined observations to nearest 1 day intervals



Latent SDE Architecture

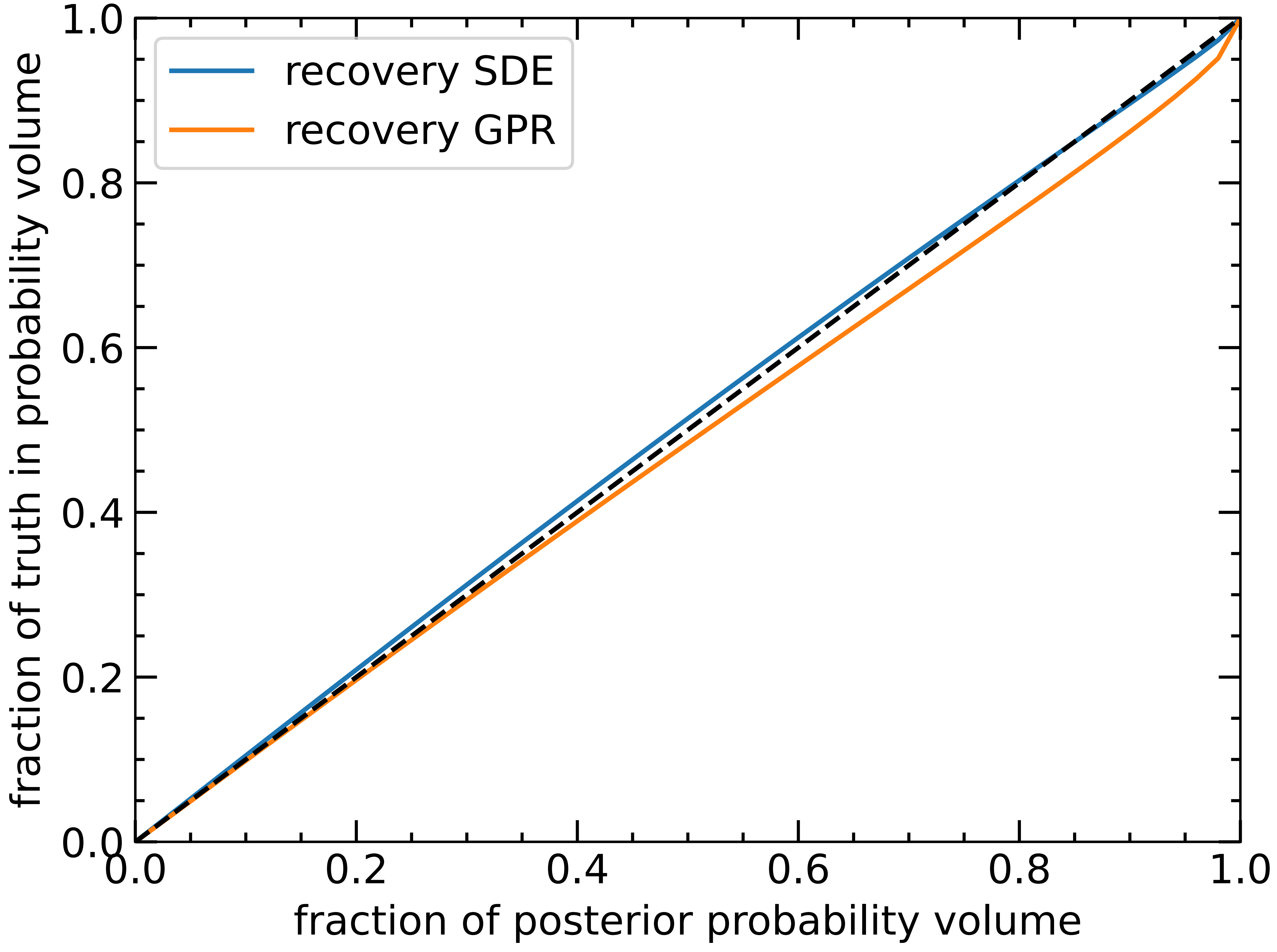
Example Light Curve Reconstruction

- Predict the mean and standard deviation of the light curve at each time step
- Compare with fixed noise, multi-output, exact Gaussian Process with Matérn-1/2 kernel

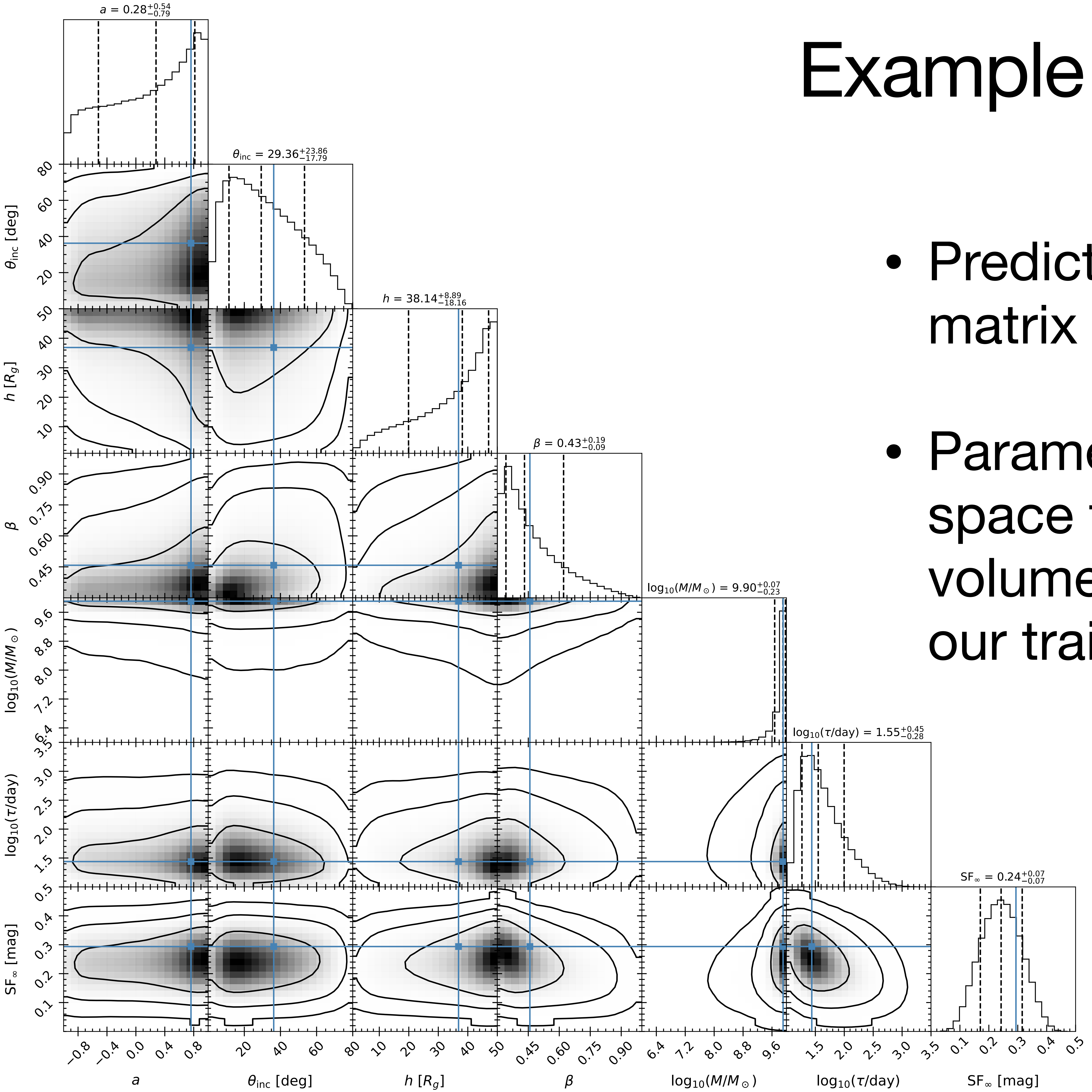
Light Curve Reconstruction Benchmark

Model	RMSE [mag]	MAE [mag]	NGLL
SDE	$\mathbf{0.0905 \pm 0.0005}$	$\mathbf{0.0616 \pm 0.0003}$	$\mathbf{-1.328 \pm 0.006}$
GPR	0.0947 ± 0.0005	0.0651 ± 0.0004	-1.200 ± 0.006

- Compare the performance of Latent SDEs to GPR on a test set of 10,000 light curves
- Latent SDEs outperform the GPR baseline across all three metrics
- Latent SDE uncertainty is also better calibrated

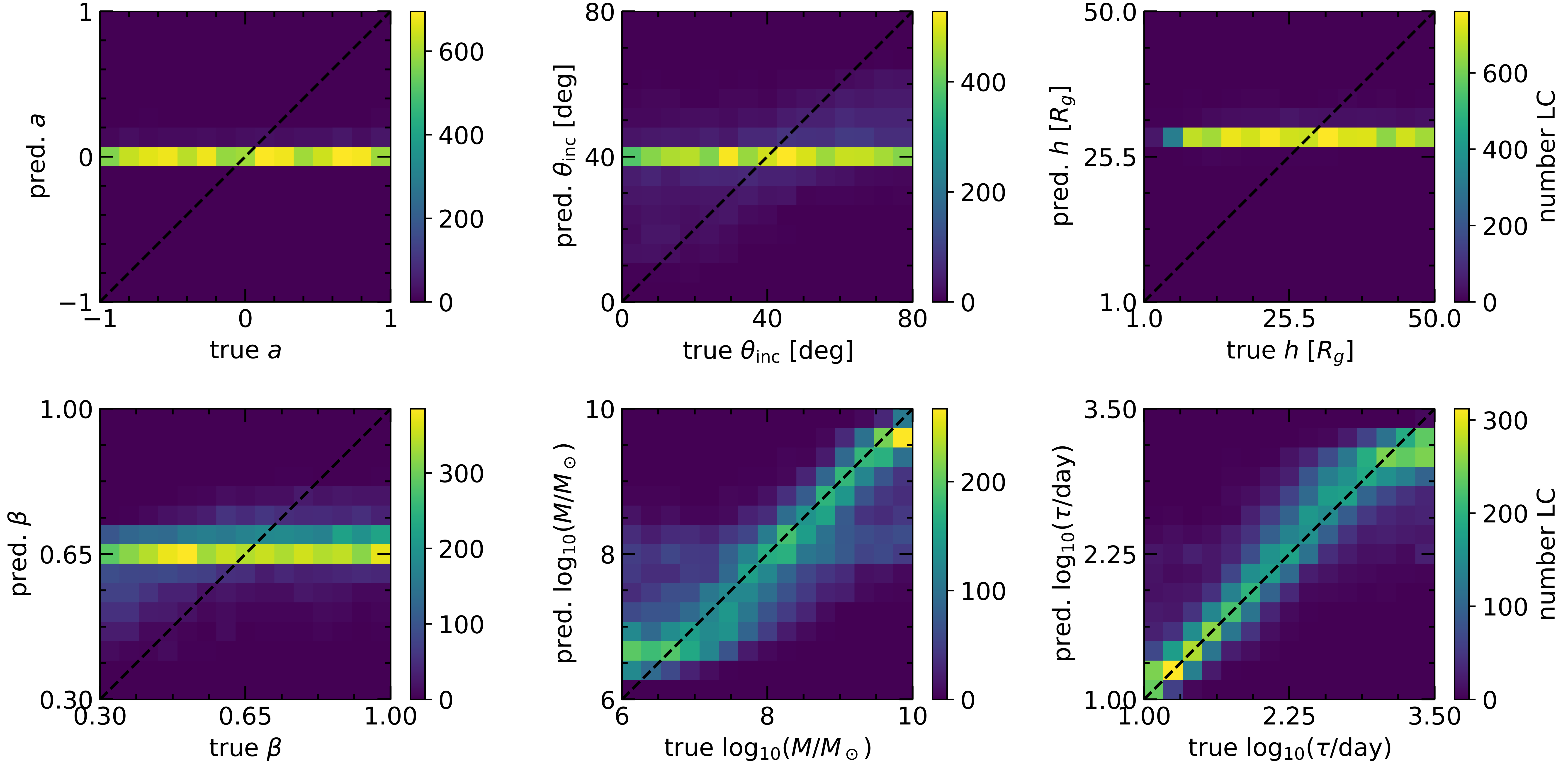


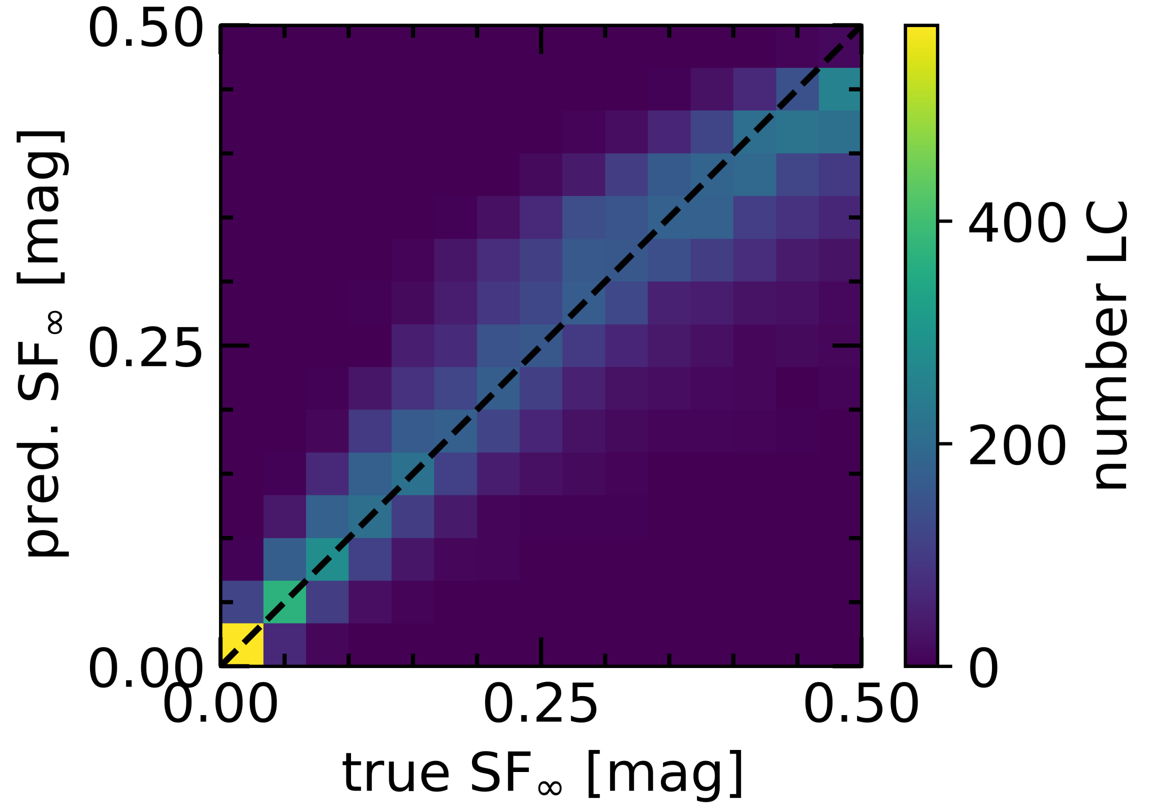
Example Parameter Predictions

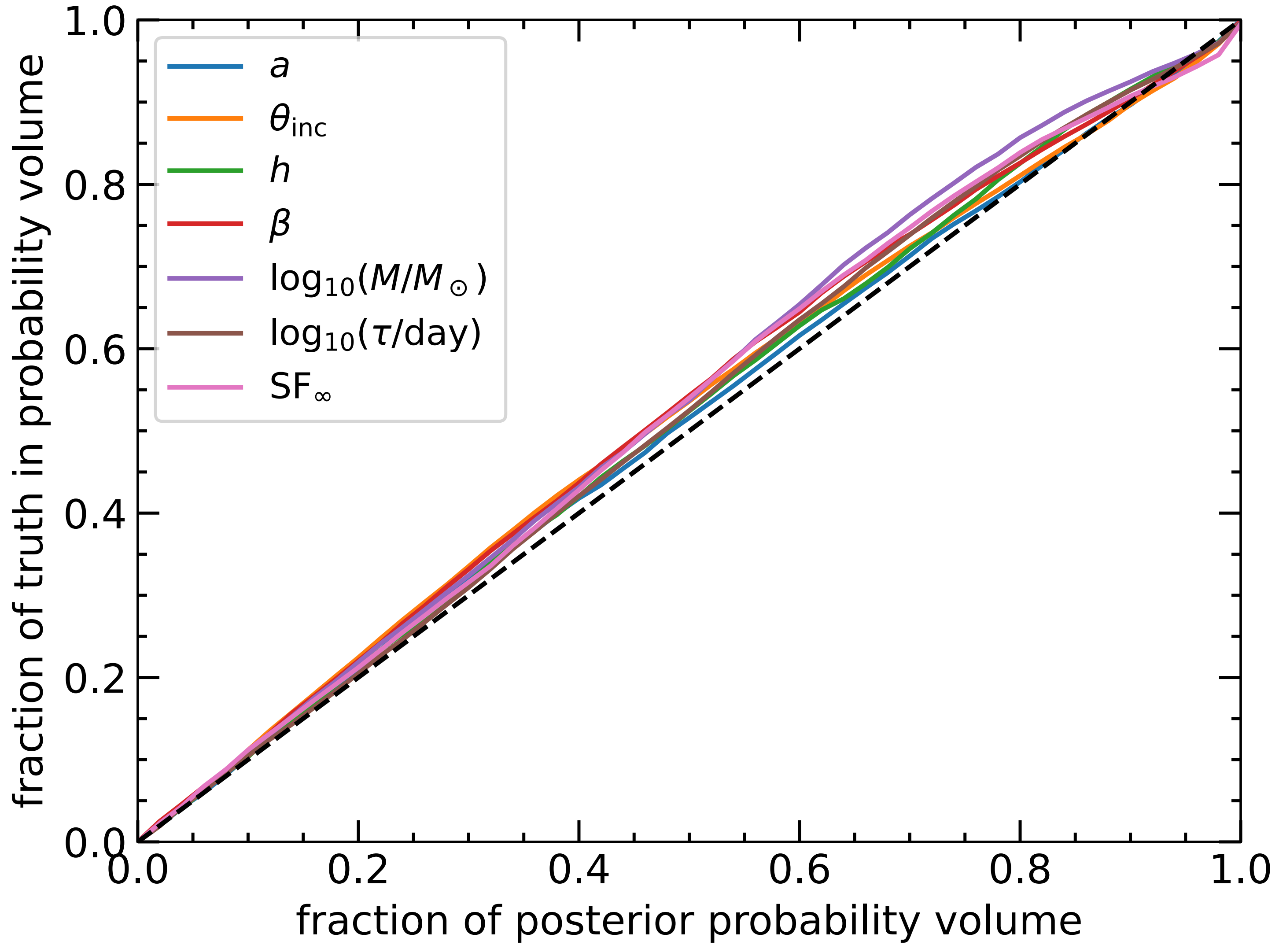


- Predict the mean and full covariance matrix for a multivariate Gaussian
- Parameterize the Gaussian in the logit space to restrict the probability volume to within the uniform range of our training set

Parameter Prediction Performance

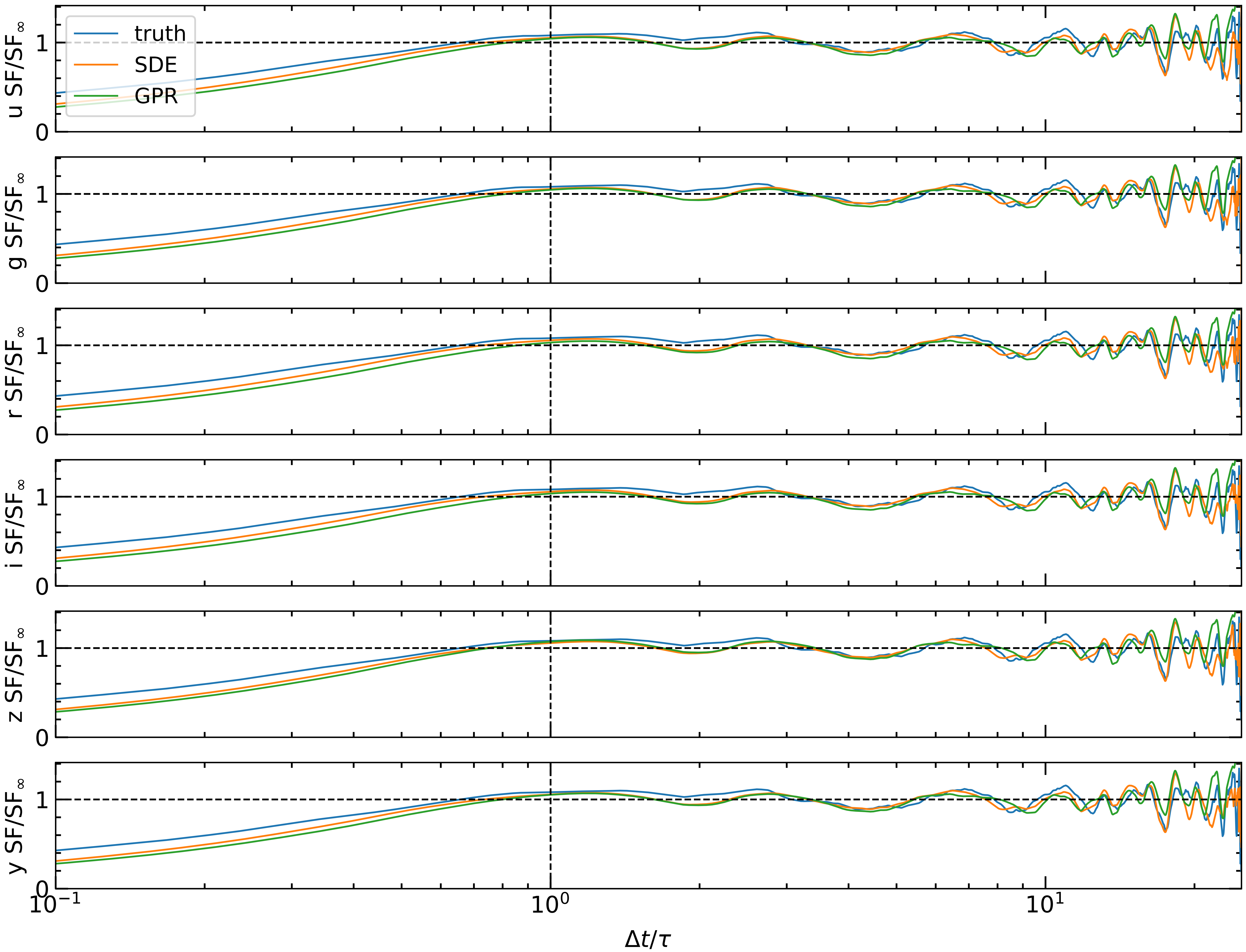






- Can consistently predict $M, \tau, \mathrm{SF}_\infty$
- For very large mass black holes we can also predict β, θ_{inc}

Example Structure Function Reconstruction



Conclusion and Outlook

- Our method is easily adaptable to any variability model and can be fine tuned to LSST light curves as data becomes available
- Can consistently predict the variability parameters and black hole mass
- Can predict the inclination angle and temperature slope when the black hole mass is very large
- Hierarchical inference of the population of black hole parameters
- Application to anomaly detection
- ML conference paper at https://arxiv.org/abs/2304.04277 and full paper and source code coming soon!

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