Current data are consistent with flat hypersurfaces in the ΛCDM but favor more lensing than the model predicts

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The lensing anomaly

It is known as the observed discrepancy between the prediction of the amount of weak lensing within the standard model and the observed one in the CMB spectra.

Some of the possible solutions:

- Non-flat models $\Omega_k \neq 0$ ($(\Omega_m + \Omega_k)h^2$).
- A phenomenological approach $A_L \neq 1$ ($C_l^{\Psi} \Rightarrow A_L C_l^{\Psi}$).

Different primordial power spectra

To analyze the CMB anisotropy data one must assume a form for the primordial power spectrum (PS).

- $P_\delta(k) = A_s \left(\frac{k}{k_0}\right)^{n_s}$ (Tilted flat PS)
- $P_\delta(q) \propto \frac{(q^2-4K^2)^2}{q(q^2-K)}$ (Untilted non-flat PS)
- $P_\delta(q) \propto \frac{(q^2-4K^2)^2}{q(q^2-K)} \left(\frac{k}{k_0}\right)^{n_s-1}$ (Planck PS)
- $P_\delta(q) \propto (q^2 - 4K)^2 |P_\zeta(A)|$ (New PS) with $A = \frac{q}{\sqrt{|K|}} - 1$ (closed) and $A = \frac{q}{\sqrt{|K|}}$ (open). [1]

where

$$k_0 = 0.05\mathrm{Mpc}^{-1} \quad q = \sqrt{k^2 + K^2} \quad K = -(H_0^2/c^2)\Omega_k$$

[1] For the details see B. Ratra PRD 106 (2022) 12, 123524.

Analytical expression for the closed new P(q)

$$\sqrt{|P_\zeta(A)|} = \left(\frac{16\pi}{m_p^2}\right)^{1/2} Q^{1/p} \frac{(2+q_s)p}{\sqrt{\pi q_s}} F(A) G(A) H(A)$$

$$F(A) = \left| -1 + \frac{W(A)}{p} \right|$$

$$G(A) = \frac{2^{-(6-4q_s+2A-W(A))/p}}{\sqrt{A}(A-1)(A+3)}$$

$$H(A) = \left| \frac{\Gamma\left(1+W(A)/p\right)\Gamma\left((2+q_s)/(2p)\right)}{\Gamma\left((2+W(A))/p\right)} \right|$$

with

$$W(A) = \sqrt{-8-4q_s+q_s^2+4A(A+2)} \quad q_s = \frac{2-2n_s}{3-n_s} \quad p = 2-q_s$$

Results obtained with Planck 2018 TT,TE,EE+lowE (P18)

Parameter	flat ΛCDM	non-flat Planck $P(q)$	non-flat new $P(q)$
Ω_m	0.3165 ± 0.0084	0.481 ± 0.062	0.444 ± 0.055
H_0[km/s/Mpc]	67.28 ± 0.61	54.5 ± 3.6	56.9 ± 3.6
Ω_k	-	-0.043 ± 0.017	-0.033 ± 0.014
χ^2_{min}	2765.80	2754.73	2757.38
ΔDIC	-	-7.34	-6.39

- Evidence in favor of closed universe $\sim 2.4\sigma$. [2]
- The non-flat models are strongly favored over the flat ΛCDM [3].

[2] W. Handley PRD 103, (2021) L041301, E. Di Valentino, A. Melchiorri and J. Silk Nature. Astron. 4, (2019) 196.

[3] $\Delta\mathrm{DIC} = \mathrm{DIC_X} - \mathrm{DIC}_{\Lambda\mathrm{CDM}}$

What if we add the A_L parameter ?

Parameter	flat ΛCDM+A_L	non-flat Planck $P(q)$+A_L	non-flat new $P(q)$+A_L
Ω_m	0.3029 ± 0.0093	0.80 ± 0.35	0.70 ± 0.43
H_0[km/s/Mpc]	68.31 ± 0.71	45 ± 11	51 ± 14
Ω_k	-	-0.130 ± 0.095	-0.10 ± 0.11
A_L	1.181 ± 0.067	0.88 ± 0.15	0.94 ± 0.20
$\chi^2_{\rm min}$	2756.12	2754.99	2756.33
ΔDIC	-5.52	-6.30	-3.10

- For the flat ΛCDM+A_L model $A_L > 1$ at 2.7σ and it is on the verge of being strongly favoured over the flat ΛCDM with $A_L = 1$.

The inclusion of the lensing data (P18+lensing)

The inclusion of the lensing data changes significantly the results

- It breaks partially the Ω_m-Ω_k-H_0-A_L degeneracy.
- The evidence in favor of closed hypersurfaces decreases until $\sim 1.5\sigma$.
- For the flat ΛCDM+A_L model $A_L = 1.073 \pm 0.041$ (1.78σ).
- None of the models is strongly favoured over the flat ΛCDM.

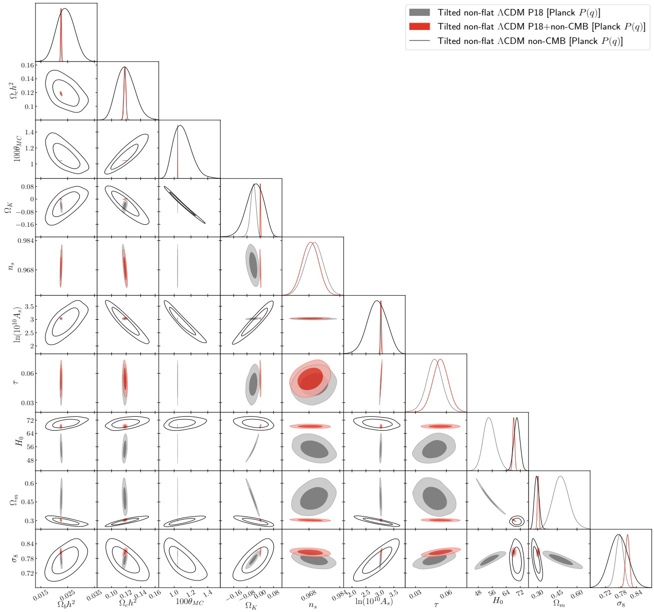
The non-CMB data compilation

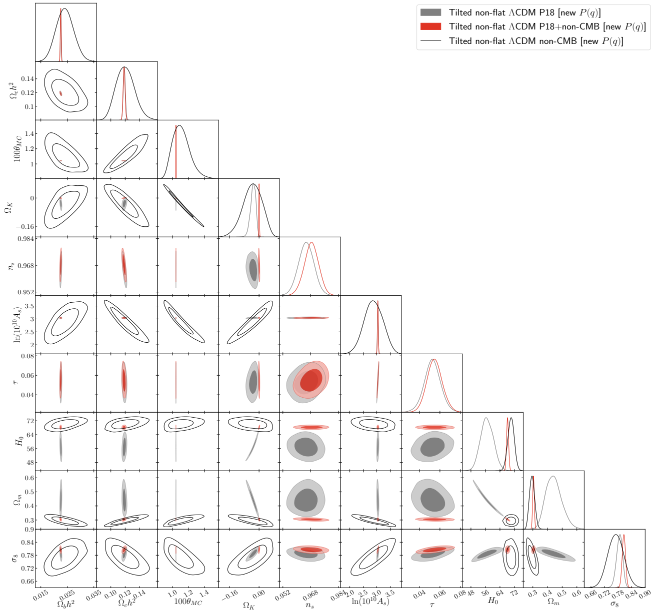
- **BAO**: 16 data points on both anisotropic and isotropic estimators in $0.122 \leq z \leq 2.334$.
- **LSS**: 8 $f\sigma_8(z_i)$ independent data points spanning the redshift range $0.02 \leq z \leq 1.36$.
- **SNIa**: 1048 for Pantheon SNIa ($0.01 < z < 2.3$) and 207 for DES 3yr SNIa ($0.015 \leq z \leq 0.7026$).
- **Cosmic Chronometers**: 31 $H(z_i)$ data points, obtained with the differential-age technique probing the range $0.070 \leq z \leq 1.965$.

Results obtained with P18+non-CMB data

Parameter	flat ΛCDM	flat ΛCDM+A_L	non-flat Planck $P(q)$	non-flat new $P(q)$
Ω_m	0.3045 ± 0.0051	0.2988 ± 0.0054	0.3040 ± 0.0055	0.3043 ± 0.0054
H_0[km/s/Mpc]	68.15 ± 0.39	68.62 ± 0.43	68.25 ± 0.56	68.21 ± 0.55
Ω_k	-	-	0.0004 ± 0.0017	0.0003 ± 0.0017
A_L	-	1.201 ± 0.061	-	-
$\chi^2_{\rm min}$	3879.35	3865.90	3878.77	3878.76
ΔDIC	-	-8.91	+2.31	+1.54

- The evidence in favor of $\Omega_k \neq 0$ has completely subsided.
- There is still evidence in favor of $A_L > 1$ at $\sim 3.3\sigma$.


Tilted non-flat ΛCDM P18 [Planck $P(q)$]
Tilted non-flat ΛCDM P18+non-CMB [Planck $P(q)$]
Tilted non-flat ΛCDM non-CMB [Planck $P(q)$]
$\Omega_b h^2$
$\Omega_c h^2$
$100\theta_{MC}$
Ω_K
n_s
$\ln(10^{10}A_s)$
τ
H_0
Ω_m
σ_8


Tilted non-flat ΛCDM P18 [new $P(q)$]
Tilted non-flat ΛCDM P18+non-CMB [new $P(q)$]
Tilted non-flat ΛCDM non-CMB [new $P(q)$]
$\Omega_b h^2$
$\Omega_c h^2$
$100\theta_{MC}$
Ω_K
n_s
$\ln(10^{10}A_s)$
τ
H_0
Ω_m
σ_8

Statistical estimators for the tensions

We can quantify the tension between two given data sets within a particular cosmological model.

$$\mathcal{I}(D_1, D_2) \equiv \exp\left(-\frac{\mathcal{G}(D_1, D_2)}{2}\right)$$ [4]

where

$$\mathcal{G}(D_1, D_2) = \mathrm{DIC}(D_1 \cup D_2) - \mathrm{DIC}(D_1) - \mathrm{DIC}(D_2)$$

Therefore $\log_{10} \mathcal{I} > 0$ when the two data sets are mutually consistent and when $\log_{10} \mathcal{I} < 0$ the two data sets are inconsistent. Applying Jeffreys' scale the level of consistency or inconsistency between the two data sets is

- substantial if $|\log_{10} \mathcal{I}| > 0.5$.
- strong if $|\log_{10} \mathcal{I}| > 1$.
- decisive if $|\log_{10} \mathcal{I}| > 2$.

[4] S. Joudaki et al. MNRAS 465 (2017) 2033.

Statistical estimators for the tensions

We can compute the probability p of two data sets being inconsistent by chance [5]

$$p = \int_{d-2log(S_D)}^{\infty} \chi_d^2(x)dx = \int_{d-2log(S_D)}^{\infty} \frac{x^{d/2-1}e^{-x/2}}{2^{d/2}\Gamma(d/2)}dx,$$

being d the Bayesian model dimensionality and S_D the suspiciousness parameter.

Considering a Gaussian analogy the value of p can be converted into a "sigma value" using

$$N_\sigma = \sqrt{2}\mathrm{Erfc}^{-1}(1-p)$$

- $p \lesssim 0.05$ ($N_\sigma = 2$) the data sets are in moderate tension.
- $p \lesssim 0.003$ ($N_\sigma = 3$) the data sets are in strong tension.

[5] W. Handley and P. Lemos PRD 100 (2019) 043504.

Data set tensions

P18 vs. non-CMB

	$\log_{10} \mathcal{I}$	N_σ
Flat ΛCDM	0.296	1.749
Flat ΛCDM$+A_L$	1.033	0.835
Non-flat Planck $P(q)$	-1.263	3.005
Non-flat new $P(q)$	-0.806	2.577

In the P18 vs. lensing case, we found less tension in the non-flat models: non-flat Planck $P(q)$, $\log_{10} \mathcal{I} = -0.486$ and $N_\sigma = 2.479$ and for the non-flat new $P(q)$, $\log_{10} \mathcal{I} = -0.062$ and $N_\sigma = 2.201$.

Conclusions

- The untilted non-flat ΛCDM is disfavoured by the Planck 2018 TT,TE,EE data.
- For the non-flat Planck $P(q)$ model a $\sim 3\sigma$ tension is found between the P18 data results and the non-CMB data results.
- The non-flat new $P(q)$ model does better than the Planck $P(q)$ model in being able to simultaneously accommodate P18 and non-CMB data as well as P18 and lensing data.
- The P18+non-CMB data favors $A_L > 1$ at $\sim 3.3\sigma$.

Thank you for your attention!