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Tensions in Cosmology: A signal of Modified Gravity?

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CANTATA

Cosmology and Astrophysics Network for
Theoretical Advances and Training Actions



- The **history** of **Astronomy, Cosmology** and **Gravity** is a **history of tensions** between **theoretical predictions** and **observations**
- **Astrophysical cosmology** has become a **precision science** with an incredibly **huge amount of data**
- **New Tensions appear.**
Are we approaching New Physics?

Aristotle - 350 BC

- According to Aristotle heavier bodies fall faster.
- Bodies fall in order to come back to their "initial state".

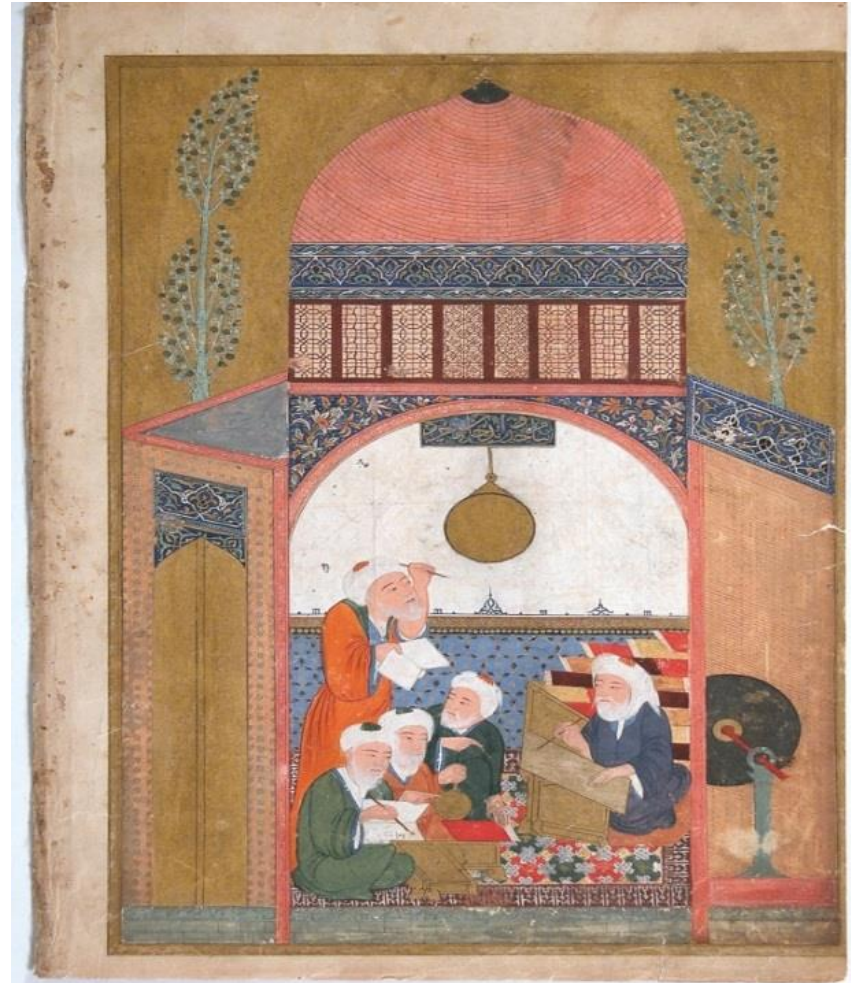


Schema huius præmissæ diuisionis Sphærarum.



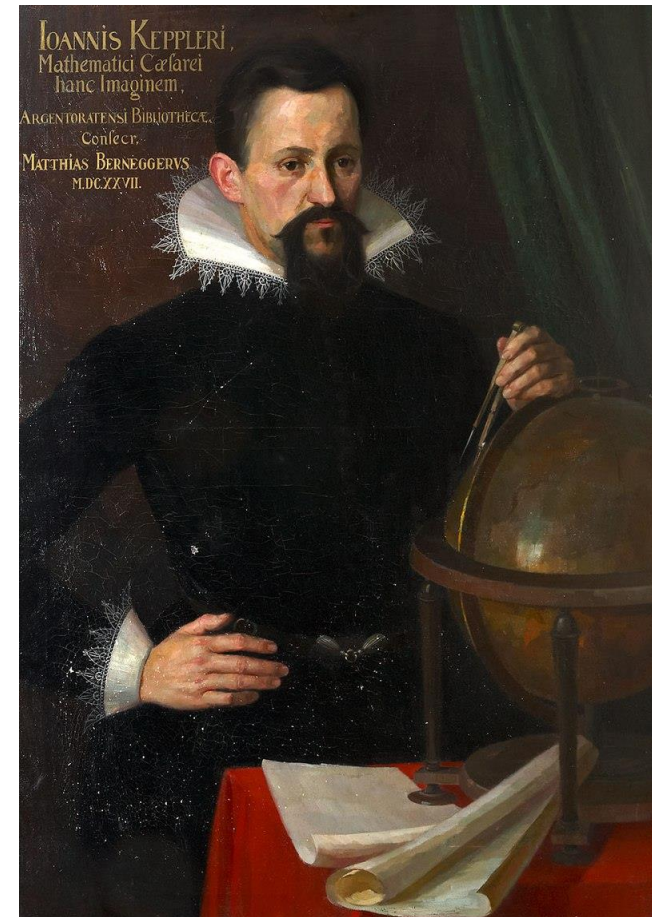
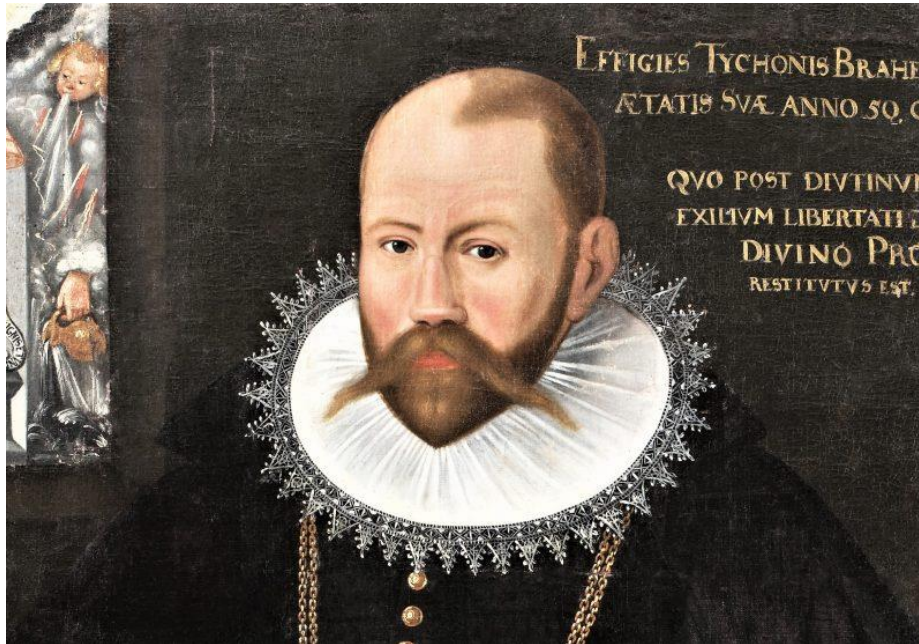
Maragha Observations

- Observations in Maragha in 11th century, started putting into doubt Earth's non-motion, however not geocentrism.



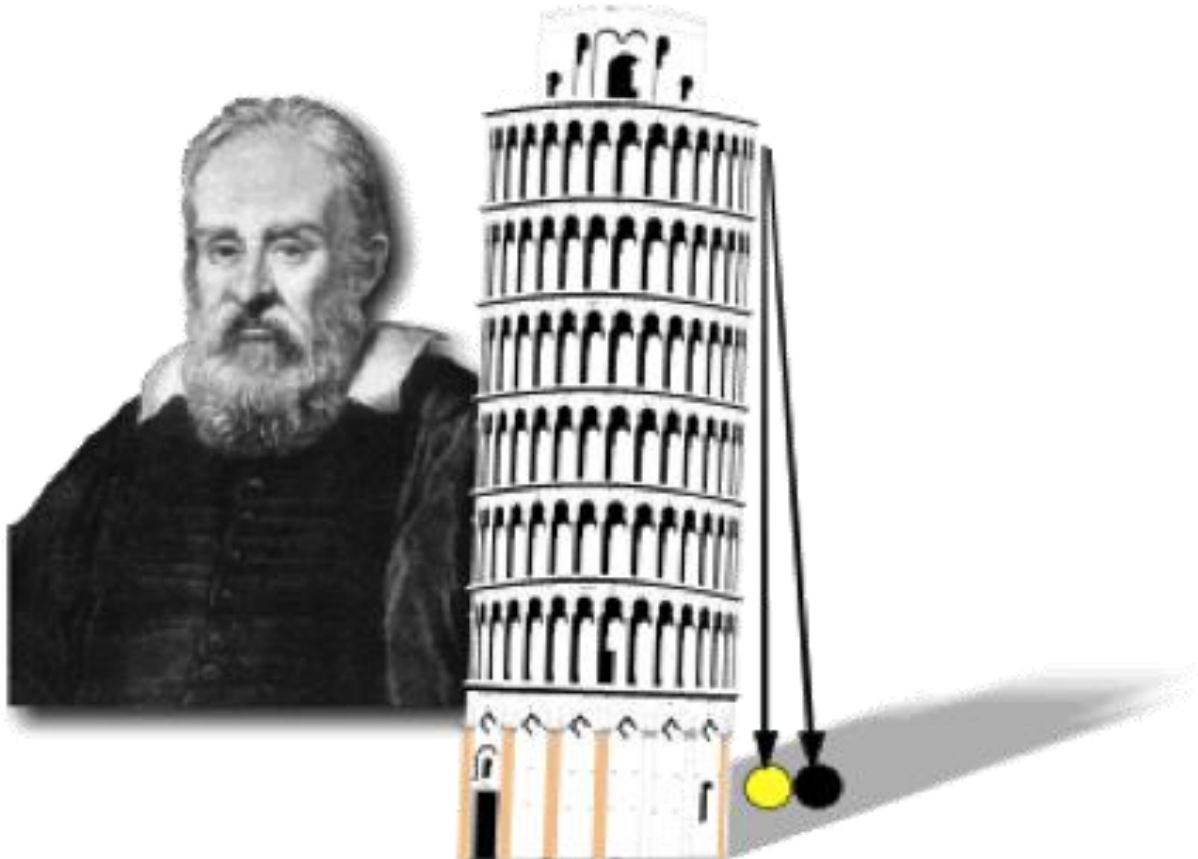
Brahe, Kepler- 1600

- Heliocentrism, elliptical Orbits



Galileo - 1600

- Bodies fall with the same speed, **independently** from their **weight**.

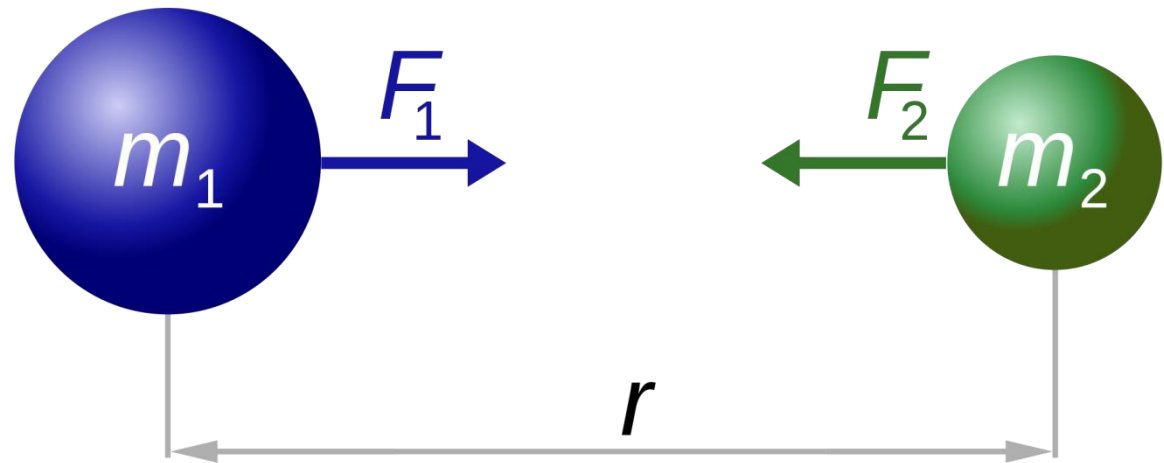
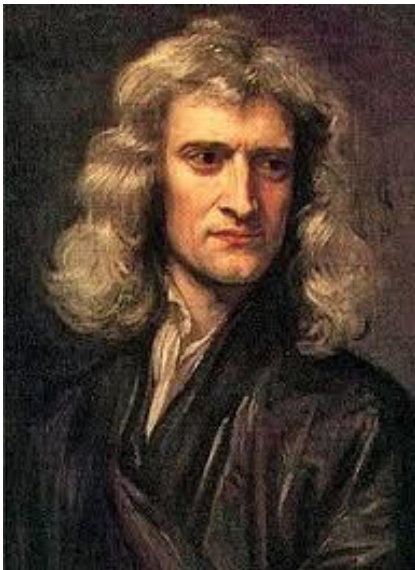


Newton - 1700

- Law of Universal Gravitation:

All bodies (either apples or planets) **attract mutually**.

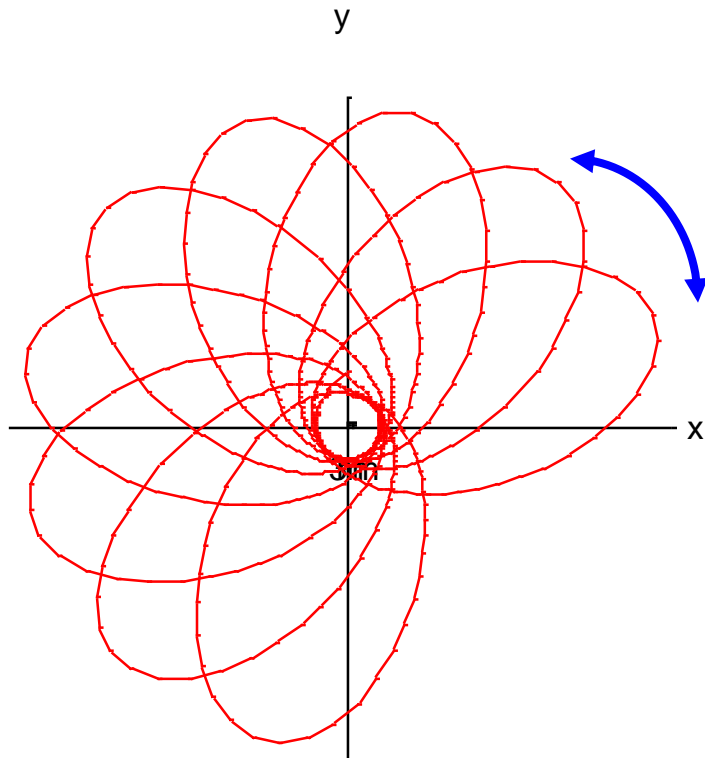
First time that **gravity is related to astronomy**



$$F_1 = F_2 = G \frac{m_1 \times m_2}{r^2}$$

Mercury perihelion - 1859

- The true orbits of planets, even if seen from the SUN are not ellipses. They are rather curves of this type:*

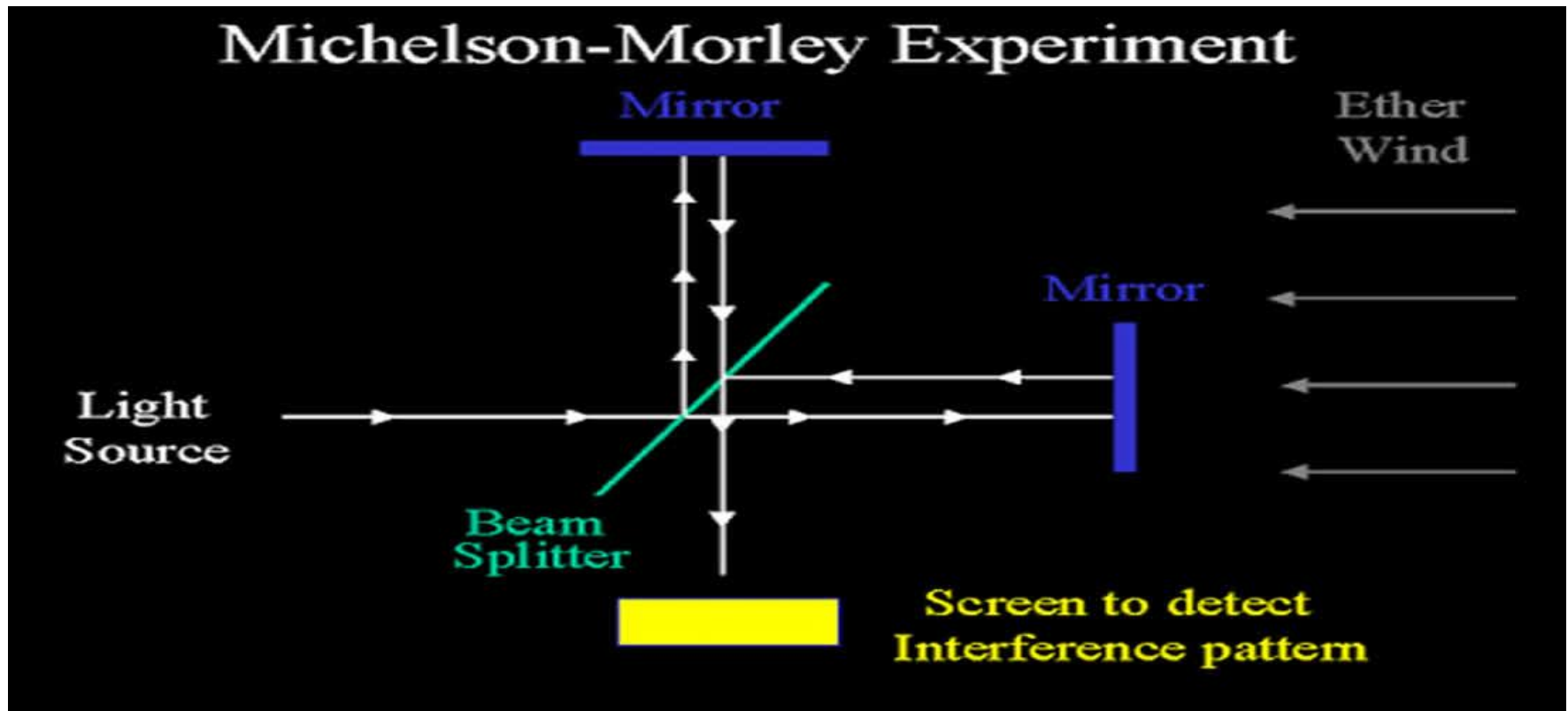


This angle is the
perihelion advance,
predicted by G.R.

For the planet Mercury it is

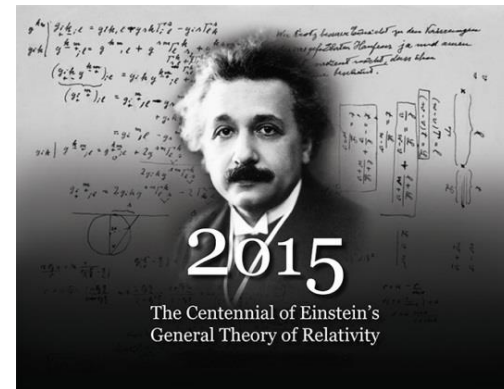
$$\Delta\varphi = 43'' \text{ of arc per century}$$

Michelson–Morley experiment - 1887



General Relativity

- Einstein 1915: **General Relativity**:



energy-momentum source of spacetime **Curvature**

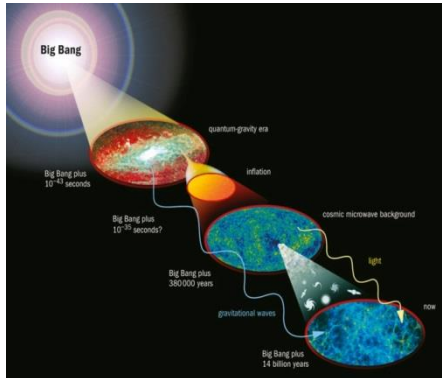
$$S = \frac{1}{16\pi G} \int d^4x \sqrt{-g} [R - 2\Lambda] + \int d^4x L_m(g_{\mu\nu}, \psi)$$

$$\Rightarrow R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R + g_{\mu\nu} \Lambda = 8\pi G T_{\mu\nu}$$

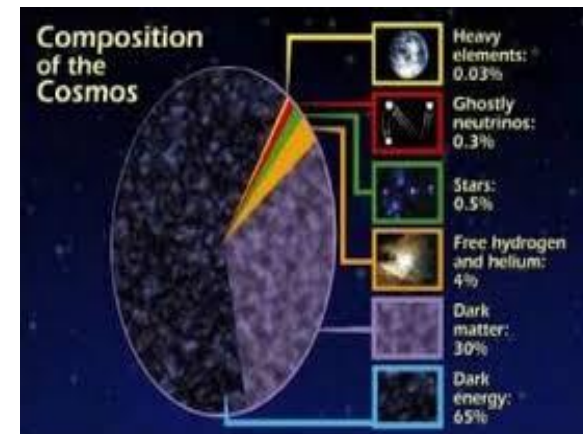
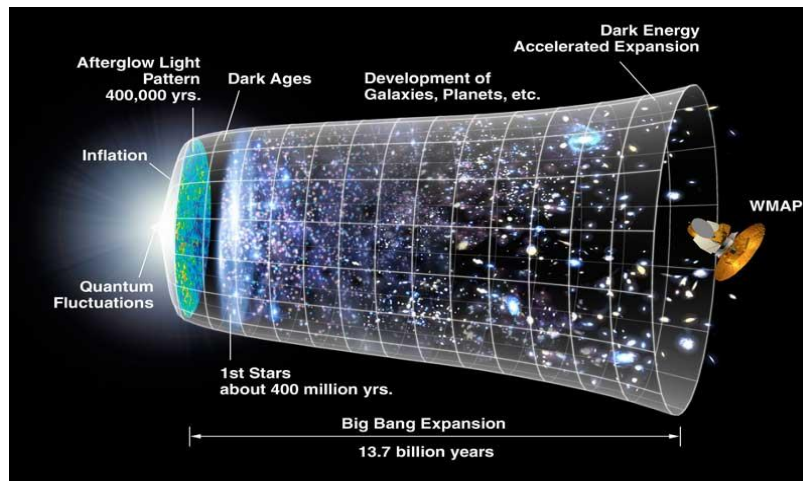
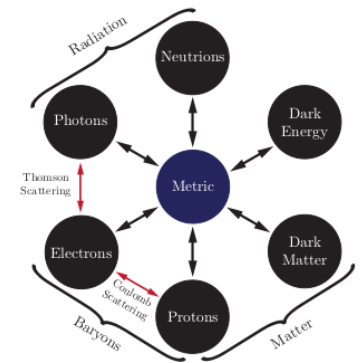
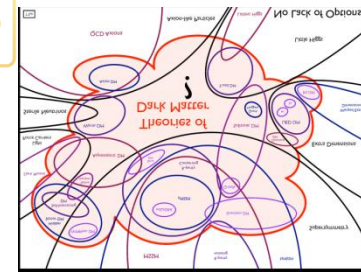
$$\text{with } T^{\mu\nu} \equiv \frac{2}{\sqrt{-g}} \frac{\delta L_m}{\delta g_{\mu\nu}}$$

Summary of 20th century Observations

The Universe history:



mass $\sim 2.3 \text{ MeV}/c^2$	$\sim 1.275 \text{ GeV}/c^2$	$\sim 173.37 \text{ GeV}/c^2$	0	$\sim 126 \text{ GeV}/c^2$
charge $\sim 2/3$	$2/3$	$2/3$	0	0
spin $\sim 1/2$	$1/2$	$1/2$	1	0
u up	c charm	t top	g gluon	H Higgs boson
$\sim 4.8 \text{ MeV}/c^2$	$\sim 55 \text{ MeV}/c^2$	$\sim 4.18 \text{ GeV}/c^2$	0	
$-1/3$	$-1/3$	$-1/3$	0	
$1/2$	$1/2$	$1/2$	1	
d down	s strange	b bottom	γ photon	
$\sim 0.511 \text{ MeV}/c^2$	$105.7 \text{ MeV}/c^2$	$1.777 \text{ GeV}/c^2$	$91.2 \text{ GeV}/c^2$	
-1	-1	-1	0	
$1/2$	$1/2$	$1/2$	1	
e electron	μ muon	τ tau	Z Z boson	
$\sim 2.2 \text{ eV}/c^2$	$\sim 0.17 \text{ MeV}/c^2$	$\sim 15.5 \text{ MeV}/c^2$	$80.4 \text{ GeV}/c^2$	
0	0	0	± 1	
$1/2$	$1/2$	$1/2$	1	
ν_e electron neutrino	ν_μ muon neutrino	ν_τ tau neutrino	W W boson	



Standard Model of Cosmology

Λ CDM Paradigm + Inflation

$$H(t)^2 + \frac{k}{a(t)^2} = \frac{8\pi G}{3} [\rho_{dm}(t) + \rho_b(t) + \rho_r(t)] + \frac{\Lambda}{3}$$

$$w_{\Lambda} \equiv \frac{p_{\Lambda}}{\rho_{\Lambda}} = -1$$

$$\dot{H}(t) - \frac{k}{a(t)^2} = -4\pi G [\rho_{dm}(t) + p_{dm}(t) + \rho_b(t) + p_b(t) + \rho_r(t) + p_r(t)]$$

Λ CDM concordance model is almost perfect!

- Describes the thermal history of the Universe at the background level
- Epochs of inflation, radiation, matter, late-time acceleration

Cosmology-background

- Homogeneity and isotropy: $ds^2 = -dt^2 + a^2(t) \left(\frac{dr^2}{1 - kr^2} + r^2 d\Omega^2 \right)$
- Background evolution (Friedmann equations) in flat space

$$H^2 = \frac{8\pi G}{3} (\rho_m + \rho_{DE})$$

$$\dot{H} = -4\pi G (\rho_m + p_m + \rho_{DE} + p_{DE}),$$

(the effective DE sector can be either Λ or any possible modification)

- One must obtain a $H(z)$ and $\Omega_m(z)$ and $w_{DE}(z)$ in agreement with observations (SNIa, BAO, CMB shift parameter, $H(z)$ etc)

Cosmology-perturbations

- **Perturbation evolution:** $\ddot{\delta} + 2H\dot{\delta} - 4\pi G_{\text{eff}} \rho \delta \approx 0$ where $\delta \equiv \delta\rho/\rho$
where $G_{\text{eff}}(z, k)$ is the **effective Newton's constant**, given by

$$\nabla^2 \phi \approx 4\pi G_{\text{eff}} \rho \delta,$$

under the scalar **metric perturbation** $ds^2 = -(1 + 2\phi)dt^2 + a^2(1 - 2\psi)d\vec{x}^2$

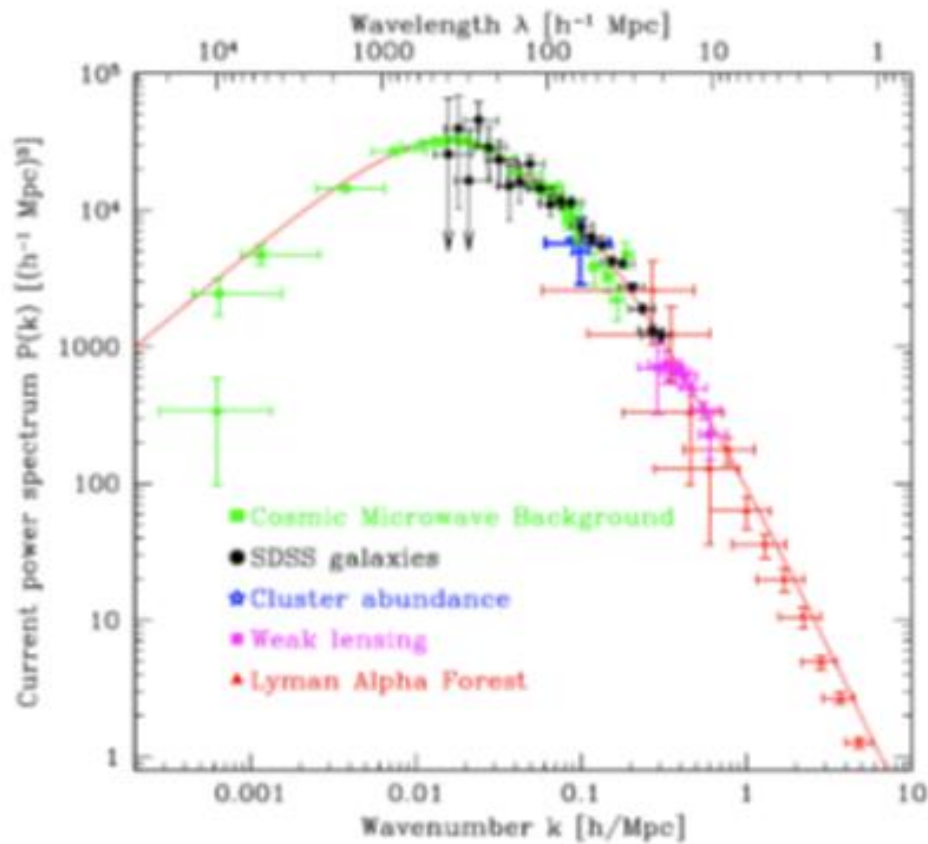
- Hence: $\delta'' + \left(\frac{(H^2)'}{2H^2} - \frac{1}{1+z} \right) \delta' \approx \frac{3}{2}(1+z) \frac{H_0^2}{H^2} \frac{G_{\text{eff}}(z, k)}{G_N} \Omega_{0m} \delta$

with $f(a) = \frac{d \ln \delta}{d \ln a}$ the **growth rate**, with $f(a) = \Omega_m(a)^{\gamma(a)}$ and $\Omega_m(a) \equiv \frac{\Omega_{0m} a^{-3}}{H(a)^2/H_0^2}$

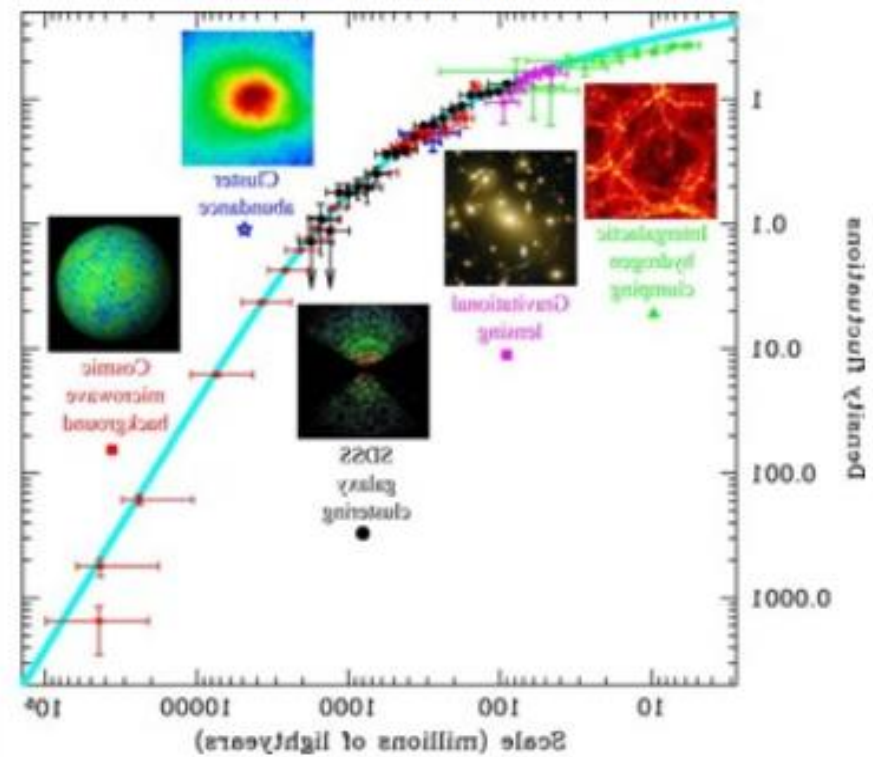
- One can define the **observable**: $f\sigma_8(a) \equiv f(a) \cdot \sigma(a) = \frac{\sigma_8}{\delta(1)} a \delta'(a)$

with $\sigma(a) = \sigma_8 \frac{\delta(a)}{\delta(1)}$ the z-dependent rms fluctuations of the linear density field within spheres of radius $R = 8h^{-1}\text{Mpc}$, and σ_8 its value today.

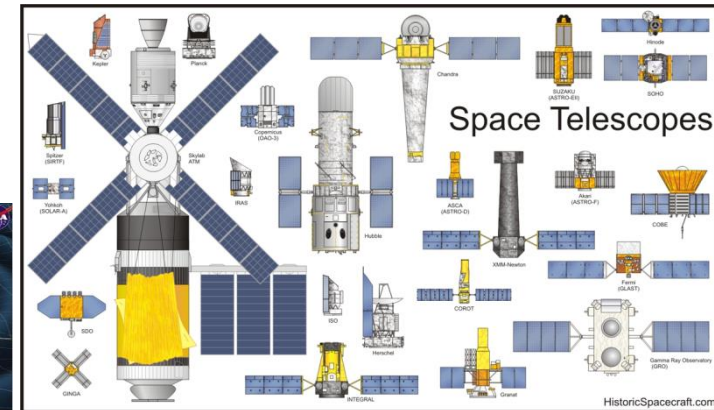
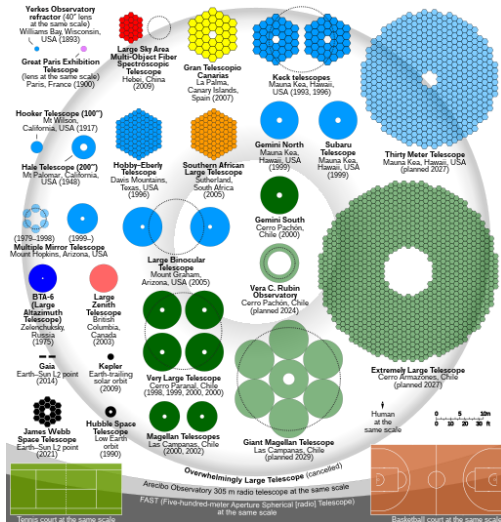
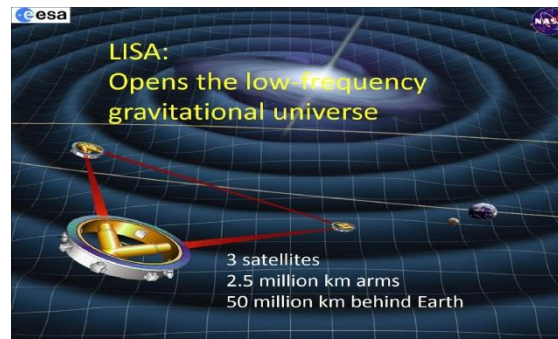
Matter Density Fluctuation Power Spectrum



A different convention:
plot $P(k)k^3$



Cosmology in the 21st century



Issues of Λ CDM Paradigm

- 1) General Relativity is non-renormalizable. It cannot get quantized.
- 2) The cosmological-constant problem.
- 3) How to describe primordial universe (inflation)
- 4) Physics of Dark Matter
- 5) A huge amount of accumulating data suggest possible tensions:

H_0 , $f\sigma_8$

Challenges for Λ CDM Beyond H_0 and S_8

- A. The A_{lens} Anomaly in the CMB Angular Power Spectrum
- B. Hints for a Closed Universe from *Planck* Data
- C. Large-Angular-Scale Anomalies in the CMB Temperature and Polarization
 - 1. The Lack of Large-Angle CMB Temperature Correlations
 - 2. Hemispherical Power Asymmetry
 - 3. Quadrupole and Octopole Anomalies
 - 4. Point-Parity Anomaly
 - 5. Variation in Cosmological Parameters Over the Sky
 - 6. The Cold Spot
 - 7. Explaining the Large-Angle Anomalies
 - 8. Predictions and Future Testability
 - 9. Summary
- D. Abnormal Oscillations of Best Fit Parameter Values
- E. Anomalously Strong ISW Effect
- F. Cosmic Dipoles
 - 1. The α Dipole
 - 2. Galaxy Cluster Anisotropies and Anomalous Bulk Flows
 - 3. Radio Galaxy Cosmic Dipole
 - 4. QSO Cosmic Dipole and Polarisation Alignments
 - 5. Dipole in SNIa
 - 6. Emergent Dipole in H_0
 - 7. CMB Dipole: Intrinsic Versus Kinematic?
- G. The Ly- α Forest BAO and CMB Anomalies
 - 1. The Ly- α Forest BAO Anomaly
 - 2. Ly- α -*Planck* 2018 Tension in n_s - Ω_m
- H. Parity Violating Rotation of CMB Linear Polarization
 - 1. The Lithium Problem
- J. Quasars Hubble Diagram Tension with Planck- Λ CDM
- K. Oscillating Force Signals in Short Range Gravity Experiments
- L. Λ CDM and the Dark Matter Phenomenon at Galactic Scales

[L. Perivolaropoulos , F. Scara, New Astron.Rev (2022), 2105.05208 [astro-ph.CO]]

H0 tension

- Tension (5σ !)** between the **data** (direct measurements) and **Planck/ Λ CDM** (indirect measurements). The data indicate **a lack of "gravitational power"**.

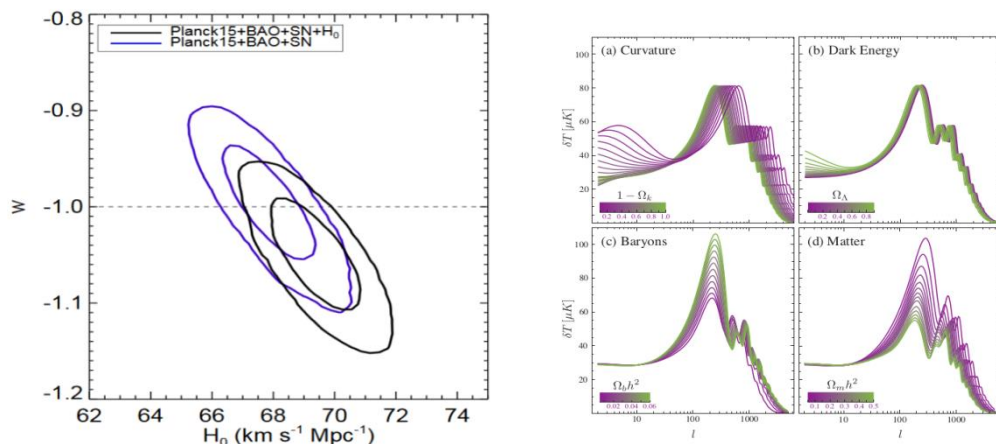
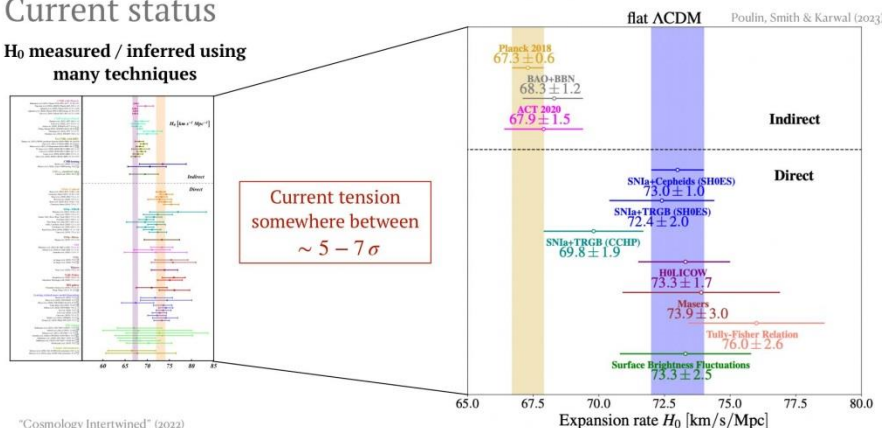


Figure 26. The CMB power spectrum as a function of cosmological parameters

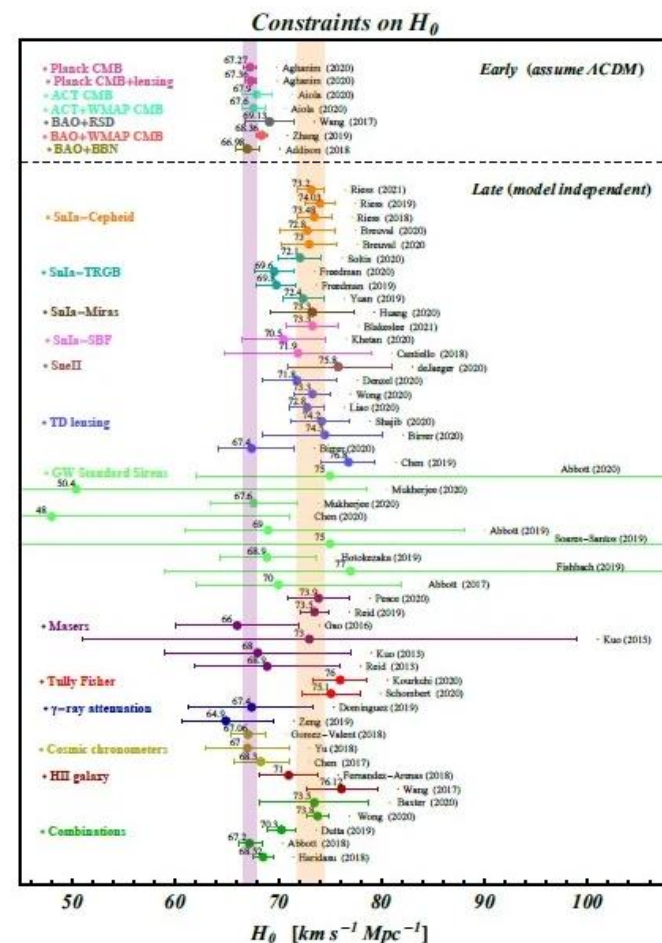
[Riess et al, Astrophys.J 826]

Current status

H_0 measured / inferred using many techniques



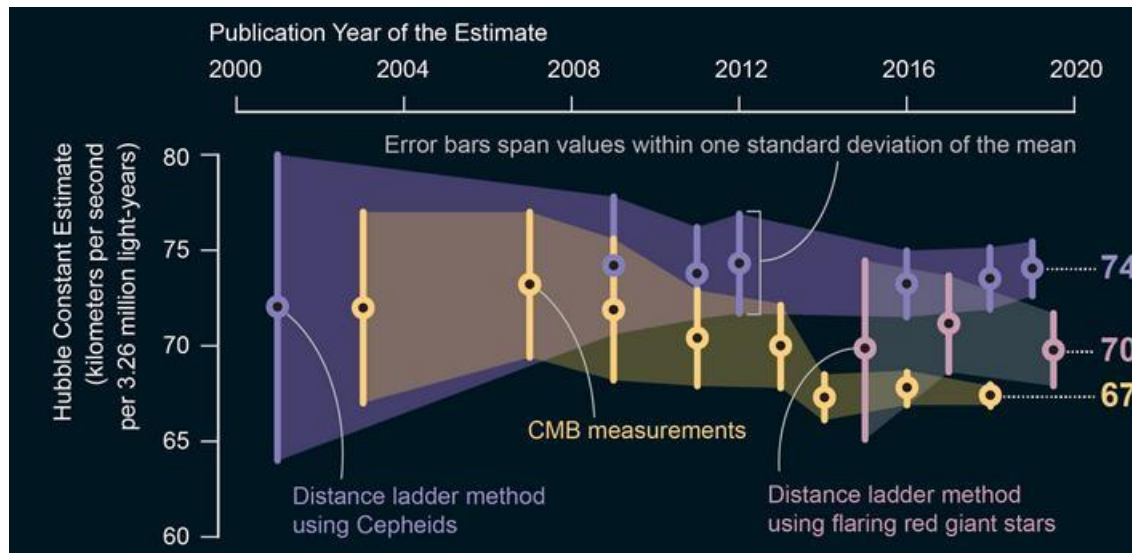
"Cosmology Intertwined" (2022)



[Abdalla et al, JHEAp (2022)] 18

H0 tension

- **Tension** between the **data** (direct measurements) and **Planck/ Λ CDM** (indirect measurements). This tension could be due to **systematics**.
- If not systematics then we may need **changes in Λ CDM** in **early** or **late** time behavior. **5σ** seems to be very serious!



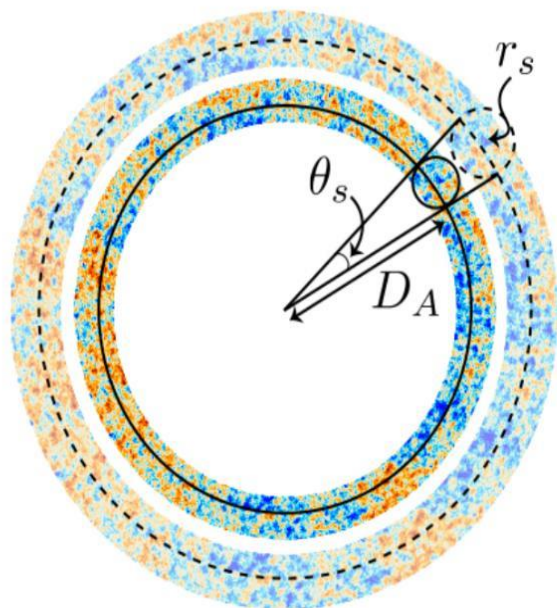
- Change early or late Universe physics. **Higher number** of effective **relativistic species**, **dynamical dark energy**, **non-zero curvature**, etc.
- The data indicate **a lack of "gravitational power"**. **Modified Gravity**.

Restoring cosmological concordance

Is LCDM Wrong?

$$\theta_s = \frac{r_s}{D_A}$$

0.04% precision



$$r_s \propto \int_0^{t_{\text{recom}}} dt \frac{c_s(t)}{\rho(t)}$$

$$D_A \propto \frac{1}{H_0} \int_{t_{\text{recom}}}^{t_{\text{today}}} dt \frac{1}{\rho(t)}$$

How do we increase H_0 ?

Decrease sound horizon (r_s)

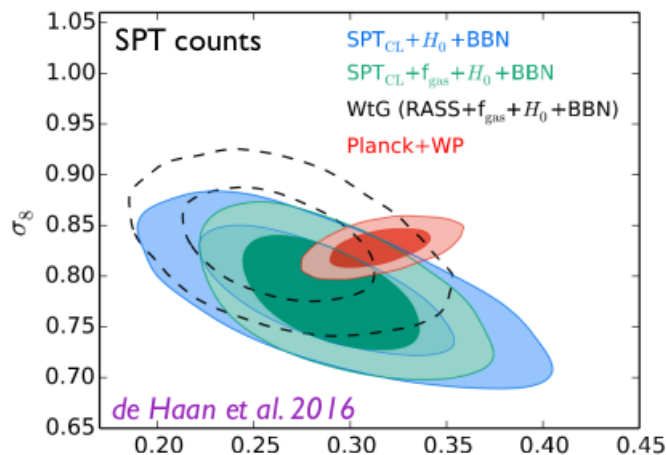
Increase integral in angular
diameter distance (D_A)

“Early time solutions”

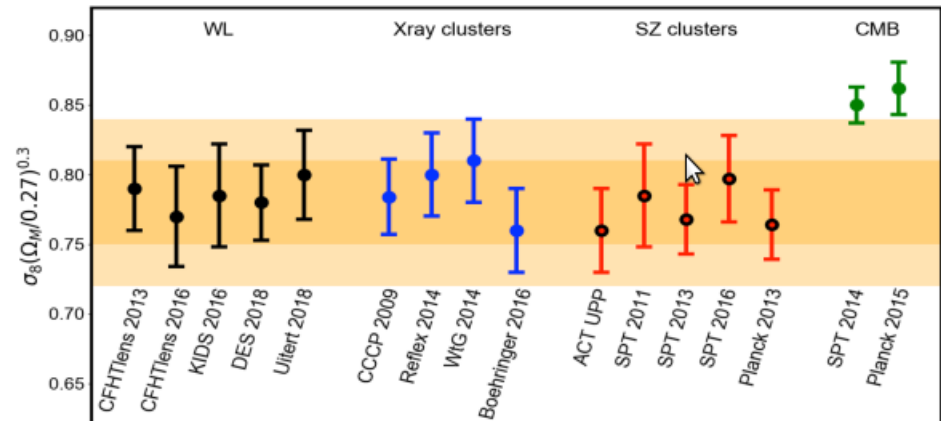
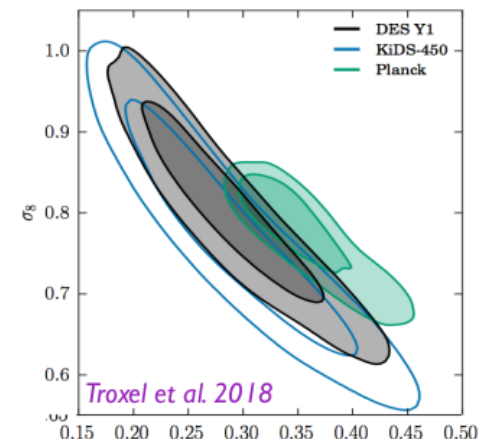
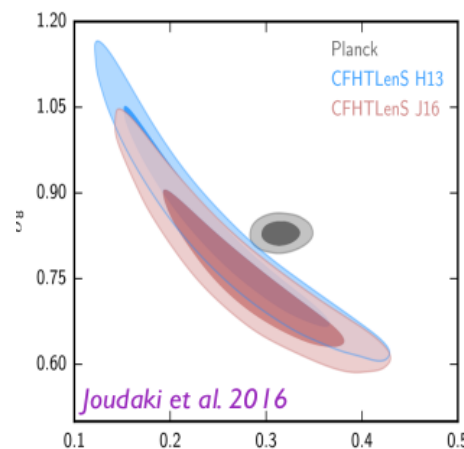
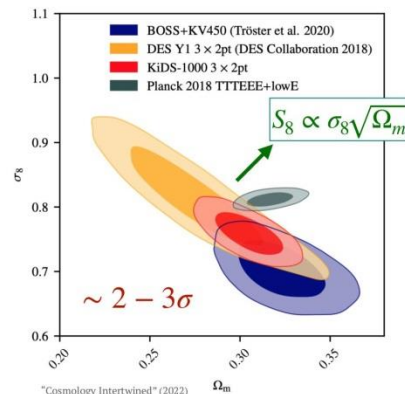
“Late time solutions”

S8 Tension

- **Tension** between **direct data** and **Planck/ Λ CDM estimation**. The data indicate **less matter clustering** in structures at intermediate-small cosmological scales.



The S8 Tension



S8 Tension

TABLE II: A compilation of RSD data that we found published from 2006 since 2018

Index	Dataset	z	$f\sigma_8(z)$	Refs.	Year	Fiducial Cosmology
1	SDSS-LRG	0.35	0.440 ± 0.050	[75]	30 October 2006	$(\Omega_{0m}, \Omega_K, \sigma_8) = (0.25, 0, 0.756)[76]$
2	VVDS	0.77	0.490 ± 0.18	[75]	6 October 2009	$(\Omega_{0m}, \Omega_K, \sigma_8) = (0.25, 0, 0.78)$
3	2dFGRS	0.17	0.510 ± 0.060	[75]	6 October 2009	$(\Omega_{0m}, \Omega_K) = (0.3, 0, 0.9)$
4	2MRS	0.02	0.314 ± 0.048	[77], [78]	13 November 2010	$(\Omega_{0m}, \Omega_K, \sigma_8) = (0.265, 0, 0.65)$
5	SnIa+IRAS	0.02	0.398 ± 0.065	[78]	20 October 2011	$(\Omega_{0m}, \Omega_K, \sigma_8) = (0.3, 0, 0.814)$
6	SDSS-LRG-200	0.25	0.3512 ± 0.0583	[80]	9 December 2011	$(\Omega_{0m}, \Omega_K, \sigma_8) = (0.276, 0, 0.8)$
7	SDSS-LRG-200	0.37	0.4602 ± 0.0378	[80]	9 December 2011	
8	SDSS-LRG-60	0.25	0.3665 ± 0.0601	[80]	9 December 2011	$(\Omega_{0m}, \Omega_K, \sigma_8) = (0.276, 0, 0.8)$
9	SDSS-LRG-60	0.37	0.4031 ± 0.0586	[80]	9 December 2011	
10	WiggleZ	0.44	0.413 ± 0.080	[46]	12 June 2012	$(\Omega_{0m}, h, \sigma_8) = (0.27, 0.71, 0.8)$
11	WiggleZ	0.60	0.390 ± 0.063	[46]	12 June 2012	$C_{ij} = Eq.(3.3)$
12	WiggleZ	0.73	0.437 ± 0.072	[46]	12 June 2012	
13	6dFGS	0.067	0.423 ± 0.055	[81]	4 July 2012	$(\Omega_{0m}, \Omega_K, \sigma_8) = (0.27, 0, 0.76)$
14	SDSS-BOSS	0.30	0.407 ± 0.055	[82]	11 August 2012	$(\Omega_{0m}, \Omega_K, \sigma_8) = (0.25, 0, 0.804)$
15	SDSS-BOSS	0.40	0.419 ± 0.041	[82]	11 August 2012	
16	SDSS-BOSS	0.50	0.427 ± 0.043	[82]	11 August 2012	
17	SDSS-BOSS	0.60	0.433 ± 0.067	[82]	11 August 2012	
18	Vipers	0.80	0.470 ± 0.080	[83]	9 July 2013	$(\Omega_{0m}, \Omega_K, \sigma_8) = (0.25, 0, 0.82)$
19	SDSS-DR7-LRG	0.35	0.429 ± 0.089	[84]	8 August 2013	$(\Omega_{0m}, \Omega_K, \sigma_8) = (0.25, 0, 0.809)[85]$
20	GAMA	0.18	0.360 ± 0.090	[86]	22 September 2013	$(\Omega_{0m}, \Omega_K, \sigma_8) = (0.27, 0, 0.8)$
21	GAMA	0.38	0.440 ± 0.060	[86]	22 September 2013	
22	BOSS-LOWZ	0.32	0.384 ± 0.095	[87]	17 December 2013	$(\Omega_{0m}, \Omega_K, \sigma_8) = (0.274, 0, 0.8)$
23	SDSS DR10 and DR11	0.32	0.48 ± 0.10	[87]	17 December 2013	$(\Omega_{0m}, \Omega_K, \sigma_8) = (0.274, 0, 0.8)[88]$
24	SDSS DR10 and DR11	0.57	0.417 ± 0.045	[87]	17 December 2013	
25	SDSS-MGS	0.15	0.490 ± 0.145	[89]	30 January 2015	$(\Omega_{0m}, h, \sigma_8) = (0.31, 0.67, 0.83)$
26	SDSS-veloc	0.10	0.370 ± 0.130	[90]	16 June 2015	$(\Omega_{0m}, \Omega_K, \sigma_8) = (0.3, 0, 0.89)[91]$
27	FastSound	1.40	0.482 ± 0.116	[92]	25 November 2015	$(\Omega_{0m}, \Omega_K, \sigma_8) = (0.27, 0, 0.82)[93]$
28	SDSS-CMASS	0.59	0.488 ± 0.060	[94]	8 July 2016	$(\Omega_{0m}, h, \sigma_8) = (0.307115, 0.6777, 0.8288)$
29	BOSS DR12	0.38	0.497 ± 0.045	[2]	11 July 2016	$(\Omega_{0m}, \Omega_K, \sigma_8) = (0.31, 0, 0.8)$
30	BOSS DR12	0.51	0.458 ± 0.038	[2]	11 July 2016	
31	BOSS DR12	0.61	0.436 ± 0.034	[2]	11 July 2016	
32	BOSS DR12	0.38	0.477 ± 0.051	[95]	11 July 2016	$(\Omega_{0m}, h, \sigma_8) = (0.31, 0.676, 0.8)$
33	BOSS DR12	0.51	0.453 ± 0.050	[95]	11 July 2016	
34	BOSS DR12	0.61	0.410 ± 0.044	[95]	11 July 2016	
35	Vipers v7	0.76	0.440 ± 0.040	[55]	26 October 2016	$(\Omega_{0m}, \sigma_8) = (0.308, 0.8149)$
36	Vipers v7	1.05	0.280 ± 0.080	[55]	26 October 2016	
37	BOSS LOWZ	0.32	0.427 ± 0.056	[96]	26 October 2016	$(\Omega_{0m}, \Omega_K, \sigma_8) = (0.31, 0, 0.8475)$
38	BOSS CMASS	0.57	0.426 ± 0.029	[96]	26 October 2016	
39	Vipers	0.727	0.296 ± 0.0765	[97]	21 November 2016	$(\Omega_{0m}, \Omega_K, \sigma_8) = (0.31, 0, 0.7)$
40	6dFGS+SnIa	0.02	0.428 ± 0.0465	[98]	29 November 2016	$(\Omega_{0m}, h, \sigma_8) = (0.3, 0.683, 0.8)$
41	Vipers	0.6	0.48 ± 0.12	[99]	16 December 2016	$(\Omega_{0m}, \Omega_b, n_s, \sigma_8) = (0.3, 0.045, 0.96, 0.831)[12]$
42	Vipers	0.86	0.48 ± 0.10	[99]	16 December 2016	
43	Vipers PDR-2	0.60	0.550 ± 0.120	[100]	16 December 2016	$(\Omega_{0m}, \Omega_b, \sigma_8) = (0.3, 0.045, 0.823)$
44	Vipers PDR-2	0.86	0.400 ± 0.110	[100]	16 December 2016	
45	SDSS DR13	0.1	0.48 ± 0.16	[101]	22 December 2016	$(\Omega_{0m}, \sigma_8) = (0.25, 0.89)[91]$
46	2MTF	0.001	0.505 ± 0.085	[102]	16 June 2017	$(\Omega_{0m}, \sigma_8) = (0.3121, 0.815)$
47	Vipers PDR-2	0.85	0.45 ± 0.11	[103]	31 July 2017	$(\Omega_b, \Omega_{0m}, h) = (0.045, 0.30, 0.8)$
48	BOSS DR12	0.31	0.469 ± 0.098	[49]	15 September 2017	$(\Omega_{0m}, h, \sigma_8) = (0.307, 0.6777, 0.8288)$
49	BOSS DR12	0.36	0.474 ± 0.097	[49]	15 September 2017	
50	BOSS DR12	0.40	0.473 ± 0.086	[49]	15 September 2017	
51	BOSS DR12	0.44	0.481 ± 0.076	[49]	15 September 2017	
52	BOSS DR12	0.48	0.482 ± 0.067	[49]	15 September 2017	
53	BOSS DR12	0.52	0.488 ± 0.065	[49]	15 September 2017	
54	BOSS DR12	0.56	0.482 ± 0.067	[49]	15 September 2017	
55	BOSS DR12	0.59	0.481 ± 0.066	[49]	15 September 2017	
56	BOSS DR12	0.64	0.486 ± 0.070	[49]	15 September 2017	
57	SDSS DR7	0.1	0.376 ± 0.038	[104]	12 December 2017	$(\Omega_{0m}, \Omega_b, \sigma_8) = (0.282, 0.046, 0.817)$
58	SDSS-IV	1.52	0.420 ± 0.076	[105]	8 January 2018	$(\Omega_{0m}, \Omega_b h^2, \sigma_8) = (0.26479, 0.02258, 0.8)$
59	SDSS-IV	1.52	0.396 ± 0.079	[106]	8 January 2018	$(\Omega_{0m}, \Omega_b h^2, \sigma_8) = (0.31, 0.022, 0.8225)$
60	SDSS-IV	0.978	0.379 ± 0.176	[107]	9 January 2018	$(\Omega_{0m}, \sigma_8) = (0.31, 0.8)$
61	SDSS-IV	1.23	0.385 ± 0.099	[107]	9 January 2018	
62	SDSS-IV	1.526	0.342 ± 0.070	[107]	9 January 2018	
63	SDSS-IV	1.944	0.364 ± 0.106	[107]	9 January 2018	

- **Model Dependence:** Distance to galaxies is not measured directly, so a cosmological model is assumed in order to infer distances (Λ CDM with different parameters).
- **Double counting:** Some data points correspond to the same sample of galaxies analyzed by different groups/methods etc.

[Kazantzidis, Perivolaropoulos, PRD97]

Tension2 – $f\sigma_8$

- **Tension** between the **data** and **Planck/ Λ CDM**.
- This tension could be due to **systematics**.
- **If not systematics**, the data **less matter clustering** in structures at intermediate-small cosmological scales (expressed as smaller Ω_m at $z < 0.6$, or smaller σ_8 , or $w_{DE} < -1$).
- It could be reconciled by a **mechanism that reduces the rate of clustering** between recombination and today: **Hot Dark Matter**, **Dark Matter that clusters differently** at small scales, or **Modified Gravity**.

Possible Solutions of H0 and S8 tensions

tension $\leq 1\sigma$ “Excellent models”	tension $\leq 2\sigma$ “Good models”	tension $\leq 3\sigma$ “Promising models”
Dark energy in extended parameter spaces [289] Dynamical Dark Energy [309] Metastable Dark Energy [314] PEDE [392, 394] Elaborated Vacuum Metamorphosis [400–402] IDE [314, 636, 637, 639, 652, 657, 661–663] Self-interacting sterile neutrinos [711] Generalized Chaplygin gas model [744] Galileon gravity [876, 882] Power Law Inflation [966] $f(\mathcal{T})$ [818]	Early Dark Energy [235] Phantom Dark Energy [11] Dynamical Dark Energy [11, 281, 309] GEDE [397] Vacuum Metamorphosis [402] IDE [314, 653, 656, 661, 663, 670] Critically Emergent Dark Energy [997] $f(\mathcal{T})$ gravity [814] Über-gravity [59] Reconstructed PPS [978]	Early Dark Energy [229] Decaying Warm DM [474] Neutrino-DM Interaction [506] Interacting dark radiation [517] Self-Interacting Neutrinos [700, 701] IDE [656] Unified Cosmologies [747] Scalar-tensor gravity [856] Modified recombination [986] Super Λ CDM [1007] Coupled Dark Energy [650]
Early Dark Energy [228, 235, 240, 250] Exponential Acoustic Dark Energy [259] Phantom Crossing [315] Late Dark Energy Transition [317] Metastable Dark Energy [314] PEDE [394] Vacuum Metamorphosis [402] Elaborated Vacuum Metamorphosis [401, 402] Sterile Neutrinos [433] Decaying Dark Matter [481] Neutrino-Majoron Interactions [509] IDE [637, 639, 657, 661] DM - Photon Coupling [685] $f(\mathcal{T})$ gravity theory [812] BD- Λ CDM [851] Über-Gravity [59] Galileon Gravity [875] Unimodular Gravity [890] Time Varying Electron Mass [990] Λ CDM [995] Ginzburg-Landau theory [996] Lorentzian Quintessential Inflation [979] Holographic Dark Energy [351]	Early Dark Energy [212, 229, 236, 263] Rock ‘n’ Roll [242] New Early Dark Energy [247] Acoustic Dark Energy [257] Dynamical Dark Energy [309] Running vacuum model [332] Bulk viscous models [340, 341] Holographic Dark Energy [350] Phantom Braneworld DE [378] PEDE [391, 392] Elaborated Vacuum Metamorphosis [401] IDE [659, 670] Interacting Dark Radiation [517] Decaying Dark Matter [471, 474] DM - Photon Coupling [686] Self-interacting sterile neutrinos [711] $f(\mathcal{T})$ gravity theory [817] Über-Gravity [871] VCDM [893] Primordial magnetic fields [992] Early modified gravity [859] Bianchi type I spacetime [999] $f(\mathcal{T})$ [818]	DE in extended parameter spaces [289] Dynamical Dark Energy [281, 309] Holographic Dark Energy [350] Swampland Conjectures [370] MEDE [399] Coupled DM - Dark radiation [534] Decaying Ultralight Scalar [538] BD- Λ CDM [852] Metastable Dark Energy [314] Self-Interacting Neutrinos [700] Dark Neutrino Interactions [716] IDE [634–636, 653, 656, 663, 669] Scalar-tensor gravity [855, 856] Galileon gravity [877, 881] Nonlocal gravity [886] Modified recombination [986] Effective Electron Rest Mass [989] Super Λ CDM [1007] Axi-Higgs [991] Self-Interacting Dark Matter [479] Primordial Black Holes [545]

Possible Solutions of H_0 and S8 tensions

Early-Time Alternative Proposed Models

1. Axion Monodromy
2. Early Dark Energy
3. Extra Relativistic Degrees of Freedom
4. Modified Recombination History
5. New Early Dark Energy

Late-Time Alternative Proposed Models

1. Bulk Viscous Models
2. Chameleon Dark Energy
3. Clustering Dark Energy
4. Diffusion Models
5. Dynamical Dark Energy
6. Emergent Dark Energy
7. Graduated Dark Energy - AdS to dS Transition in the Late Universe
8. Holographic Dark Energy
9. Interacting Dark Energy
10. Quintessence Models and their Various Extensions
11. Running Vacuum Models
12. Time-Varying Gravitational Constant
13. Vacuum Metamorphosis

Modified Gravity Models

1. Effective Field Theory Approach to Dark Energy and Modified Gravity
2. $f(T)$ Gravity
3. Horndeski Theory
4. Quantum Conformal Anomaly Effective Theory and Dynamical Vacuum Energy
5. Ultra-Late Time Gravitational Transitions

Beyond the FLRW Framework

1. Cosmological Fitting and Averaging Problems
2. Data Analysis in an Universe with Structure: Accounting for Regional Inhomogeneity and Anisotropy
3. Local Void Scenario

Specific Solutions Assuming FLRW

1. Active and Sterile Neutrinos
2. Cannibal Dark Matter
3. Decaying Dark Matter
4. Dynamical Dark Matter
5. Extended Parameter Spaces Involving A_{lens}
6. Cosmological Scenario with Features in the Primordial Power Spectrum
7. Interacting Dark Matter
8. Quantum Landscape Multiverse
9. Quantum Fisher Cosmology
10. Quartessence
11. Scaling Symmetry and a Mirror Sector
12. Self-Interacting Neutrinos
13. Self-Interacting Sterile Neutrinos
14. Soft Cosmology
15. Two-Body Decaying Cold Dark Matter into Dark Radiation and Warm Dark Matter

**Cosmology Intertwined:
A Review of the Particle Physics, Astrophysics, and Cosmology
Associated with the Cosmological Tensions and Anomalies**

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10 commandments for Hubble hunters

- ① I am $H_0 \approx 74$ thy Goal
- ② Thou shalt not fail to fit key data (BAO, SNela, polarization)...
- ③ ...or include a local H_0 prior in vain
- ④ Remember to not just blow up the uncertainty on H_0 ...
- ⑤ ...honour its central value, and keep an eye on your $\Delta\chi^2$ /Bayesian evidence
- ⑥ Thou shalt not murder σ_8/S_8 ...
- ⑦ ...but aim to solve this and other tensions/puzzles at the same time
- ⑧ Thy solution shall come from a compelling particle/gravity model...
- ⑨ ...which makes verifiable predictions...
- ⑩ ...which later better be verified!



Credits: Gustave Doré

Efficient model independent requirements to solve the tensions

- In general, to avoid the H_0 tension one needs a positive correction to the first Friedmann equation at late times that could yield an increase in H_0 compared to the Λ CDM scenario.

Efficient model independent requirements to solve the tensions

- For the σ_8 tension, we recall that in any cosmological model, at sub-Hubble scales and through matter epoch, the equation that governs the evolution of matter perturbations in the linear regime is

$$\ddot{\delta} + 2H\dot{\delta} = 4\pi G_{\text{eff}}\rho_m\delta, \quad (1)$$

where G_{eff} is the effective gravitational coupling given by a generalized Poisson equation.

- Solving for $\delta(a)$ provides the observable quantity $f\sigma_8(a)$, following the definitions $f(a) \equiv d \ln \delta(a)/d \ln a$ and $\sigma(a) = \sigma_8 \delta(1)/\delta(a=1)$. Hence, alleviation of the σ_8 tension may be obtained if G_{eff} becomes smaller than G_N during the growth of matter perturbations and/or if the “friction” term in (1) increases.

Efficient model independent requirements to solve the tensions

We consider a correction in the first Friedmann equation of the form

$$H(z) = -\frac{d(z)}{4} + \sqrt{\frac{d^2(z)}{16} + H_{\Lambda\text{CDM}}^2(z)}, \quad (2)$$

where $H_{\Lambda\text{CDM}}(z) \equiv H_0 \sqrt{\Omega_m(1+z)^3 + \Omega_\Lambda}$ is the Hubble rate in ΛCDM , with $\Omega_m = \rho_m/(3M_p^2 H^2)$ the matter density parameter and primes denote derivatives with respect to z .

- If $d < 0$ and is suitably chosen, one can have $H(z \rightarrow z_{\text{CMB}}) \approx H_{\Lambda\text{CDM}}(z \rightarrow z_{\text{CMB}})$ but $H(z \rightarrow 0) > H_{\Lambda\text{CDM}}(z \rightarrow 0)$; i.e., the H_0 tension is solved [one should choose $|d(z)| < H(z)$, and thus, since $H(z)$ decreases for smaller z , the deviation from ΛCDM will be significant only at low redshift].
- Since the friction term in (1) increases, the growth of structure gets damped, and therefore, the σ_8 tension is also solved.

General Relativity

Assumptions and Considerations

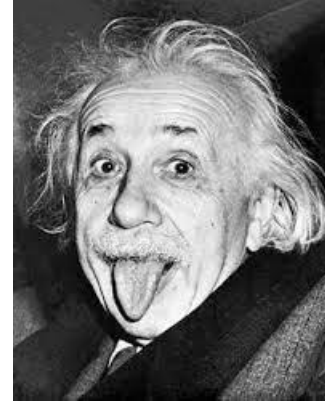
$$S = \frac{1}{16\pi G} \int d^4x \sqrt{-g} [R - 2\Lambda] + \int d^4x L_m(g_{\mu\nu}, \psi)$$

- Diffeomorphism invariance
- Spacetime dimensionality=4
- **Geometry=Curvature** (connection=Levi Civita)
- Linear in Ricci scalar
- **Metric compatibility** (zero non-metricity)
- Minimal matter coupling
- Equivalence principle
- Lorentz invariance
- Locality

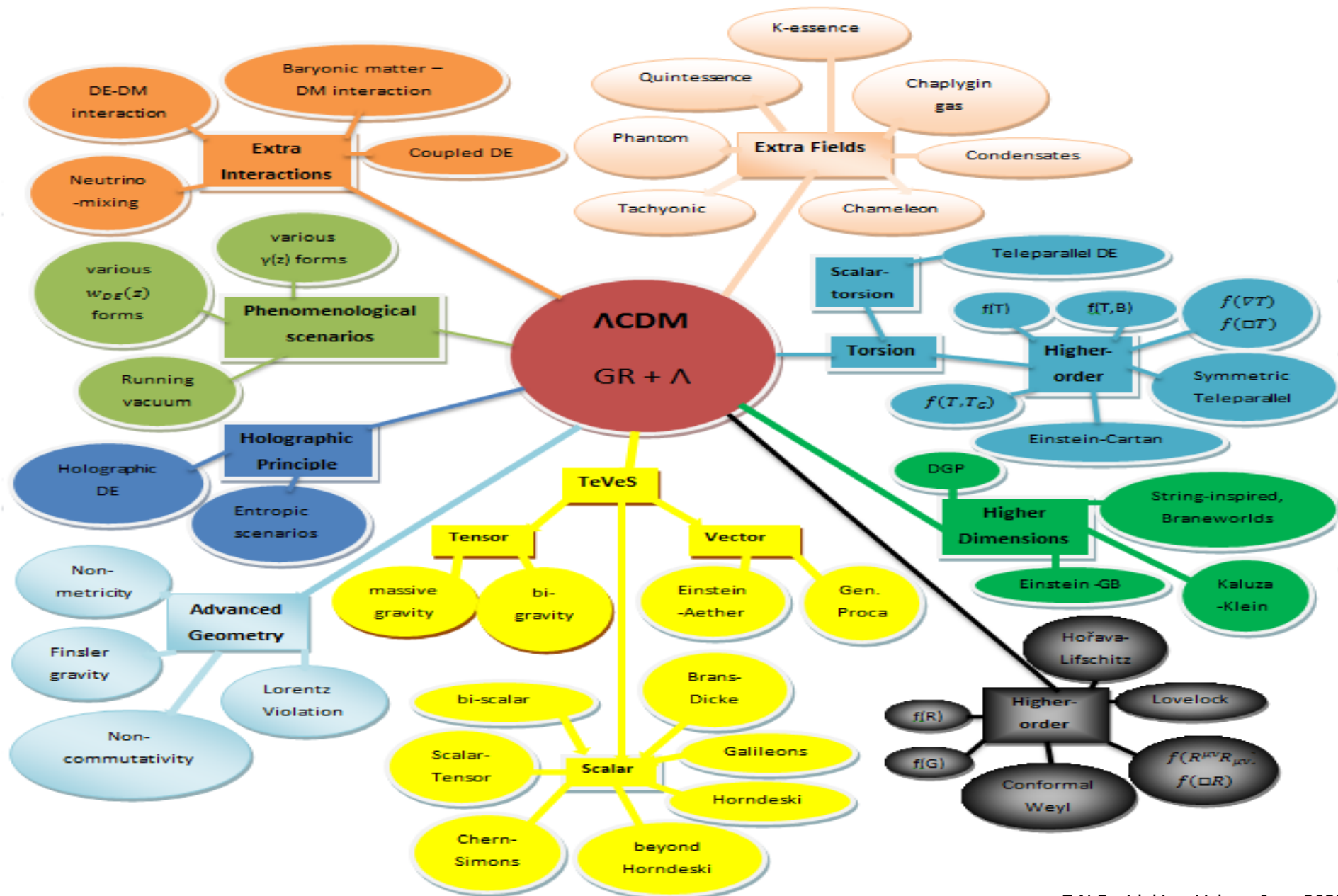
Standard Model vs General Relativity Lagrangians

$$\begin{aligned}
 & -\frac{1}{2}\partial_\nu g_\mu^a \partial_\nu g_\mu^a - g_s f^{abc} \partial_\mu g_\nu^a g_\mu^b g_\nu^c - \frac{1}{4}g_s^2 f^{abc} f^{ade} g_\mu^b g_\nu^c g_\mu^d g_\nu^e + \\
 & \frac{1}{2}ig_s^2(\bar{q}_i^\sigma \gamma^\mu q_j^\sigma)g_\mu^a + \bar{G}^a \partial^2 G^a + g_s f^{abc} \partial_\mu \bar{G}^a G^b g_\mu^c - \partial_\nu W_\mu^+ \partial_\nu W_\mu^- - \\
 & M^2 W_\mu^+ W_\mu^- - \frac{1}{2}\partial_\nu Z_\mu^0 \partial_\nu Z_\mu^0 - \frac{1}{2c_w^2}M^2 Z_\mu^0 Z_\mu^0 - \frac{1}{2}\partial_\mu A_\nu \partial_\mu A_\nu - \frac{1}{2}\partial_\mu H \partial_\mu H - \\
 & \frac{1}{2}m_h^2 H^2 - \partial_\mu \phi^+ \partial_\mu \phi^- - M^2 \phi^+ \phi^- - \frac{1}{2}\partial_\mu \phi^0 \partial_\mu \phi^0 - \frac{1}{2c_w^2}M\phi^0 \phi^0 - \beta_h \frac{[2M^2]}{g^2} + \\
 & \frac{2M}{g}H + \frac{1}{2}(H^2 + \phi^0 \phi^0 + 2\phi^+ \phi^-) + \frac{2M^4}{g^2}\alpha_h - ig_{cw}[\partial_\nu Z_\mu^0(W_\mu^+ W_\nu^- - \\
 & W_\nu^+ W_\mu^-) - Z_\nu^0(W_\mu^+ \partial_\nu W_\mu^- - W_\mu^+ \partial_\nu W_\mu^-) + Z_\mu^0(W_\nu^+ \partial_\nu W_\mu^- - \\
 & W_\nu^+ \partial_\nu W_\mu^-)] - ig_{sw}[\partial_\nu A_\mu(W_\mu^+ W_\nu^- - W_\nu^+ W_\mu^-) - A_\nu(W_\mu^+ \partial_\nu W_\mu^- - \\
 & W_\mu^+ \partial_\nu W_\mu^-) + A_\mu(W_\nu^+ \partial_\nu W_\mu^- - W_\nu^+ \partial_\nu W_\mu^-)] - \frac{1}{2}g^2 W_\mu^+ W_\mu^- W_\nu^+ W_\nu^- + \\
 & \frac{1}{2}g^2 W_\mu^+ W_\nu^- W_\mu^- W_\nu^- + g^2 c_w^2(Z_\mu^0 W_\mu^+ Z_\nu^0 W_\nu^- - Z_\mu^0 Z_\nu^0 W_\mu^+ W_\nu^-) + \\
 & g^2 s_w^2(A_\mu W_\mu^+ A_\nu W_\nu^- - A_\mu A_\nu W_\mu^+ W_\nu^-) + g^2 s_w c_w[A_\mu Z_\nu^0(W_\mu^+ W_\nu^- - \\
 & W_\nu^+ W_\mu^-) - 2A_\mu Z_\mu^0 W_\nu^+ W_\nu^-] - g\alpha[H^3 + H\phi^0 \phi^0 + 2H\phi^+ \phi^-] - \\
 & \frac{1}{8}g^2 \alpha_h[H^4 + (\phi^0)^4 + 4(\phi^+ \phi^-)^2 + 4(\phi^0)^2 \phi^+ \phi^- + 4H^2 \phi^+ \phi^- + 2(\phi^0)^2 H^2] - \\
 & gMW_\mu^+ W_\mu^- H - \frac{1}{2}g\frac{M}{c_w^2}Z_\mu^0 Z_\mu^0 H - \frac{1}{2}ig[W_\mu^+(\phi^0 \partial_\mu \phi^- - \phi^- \partial_\mu \phi^0) - \\
 & W_\mu^-(\phi^0 \partial_\mu \phi^+ - \phi^+ \partial_\mu \phi^0)] + \frac{1}{2}g[W_\mu^+(H\partial_\mu \phi^- - \phi^- \partial_\mu H) - W_\mu^-(H\partial_\mu \phi^+ - \\
 & \phi^+ \partial_\mu H)] + \frac{1}{2}g\frac{1}{c_w}(Z_\mu^0(H\partial_\mu \phi^0 - \phi^0 \partial_\mu H) - ig\frac{s_w^2}{c_w}M Z_\mu^0(W_\mu^+ \phi^- - W_\mu^- \phi^+) + \\
 & ig_{sw}MA_\mu(W_\mu^+ \phi^- - W_\mu^- \phi^+) - ig\frac{1-2c_w^2}{2c_w}Z_\mu^0(\phi^+ \partial_\mu \phi^- - \phi^- \partial_\mu \phi^+) + \\
 & ig_{sw}A_\mu(\phi^+ \partial_\mu \phi^- - \phi^- \partial_\mu \phi^+) - \frac{1}{4}g^2 W_\mu^+ W_\mu^- [H^2 + (\phi^0)^2 + 2\phi^+ \phi^-] - \\
 & \frac{1}{4}g^2 \frac{1}{c_w^2}Z_\mu^0 Z_\mu^0 [H^2 + (\phi^0)^2 + 2(2s_w^2 - 1)^2 \phi^+ \phi^-] - \frac{1}{2}g^2 \frac{s_w^2}{c_w}Z_\mu^0 \phi^0(W_\mu^+ \phi^- + \\
 & W_\mu^- \phi^+) - \frac{1}{2}ig^2 \frac{s_w^2}{c_w}Z_\mu^0 H(W_\mu^+ \phi^- - W_\mu^- \phi^+) + \frac{1}{2}g^2 s_w A_\mu \phi^0(W_\mu^+ \phi^- + \\
 & W_\mu^- \phi^+) + \frac{1}{2}ig^2 s_w A_\mu H(W_\mu^+ \phi^- - W_\mu^- \phi^+) - g^2 \frac{s_w}{c_w}(2c_w^2 - 1)Z_\mu^0 A_\mu \phi^+ \phi^- - \\
 & g^1 s_w^2 A_\mu A_\mu \phi^+ \phi^- - \bar{e}^\lambda (\gamma^\mu \partial + m_e^\lambda) e^\lambda - \bar{\nu}^\lambda \gamma^\mu \partial \nu^\lambda - \bar{u}_j^\lambda (\gamma^\mu \partial + m_u^\lambda) u_j^\lambda - \\
 & \bar{d}_j^\lambda (\gamma^\mu \partial + m_d^\lambda) d_j^\lambda + ig_{sw}A_\mu [-(\bar{e}^\lambda \gamma^\mu e^\lambda) + \frac{2}{3}(\bar{u}_j^\lambda \gamma^\mu u_j^\lambda) - \frac{1}{3}(\bar{d}_j^\lambda \gamma^\mu d_j^\lambda)] + \\
 & \frac{ig}{4c_w}Z_\mu^0 [(\bar{\nu}^\lambda \gamma^\mu (1 + \gamma^5) \nu^\lambda) + (\bar{e}^\lambda \gamma^\mu (4s_w^2 - 1 - \gamma^5) e^\lambda) + (\bar{u}_j^\lambda \gamma^\mu (\frac{4}{3}s_w^2 - \\
 & 1 - \gamma^5) u_j^\lambda) + (\bar{d}_j^\lambda \gamma^\mu (1 - \frac{8}{3}s_w^2 - \gamma^5) d_j^\lambda)] + \frac{ig}{2\sqrt{2}}W_\mu^+ [(\bar{\nu}^\lambda \gamma^\mu (1 + \gamma^5) e^\lambda) + \\
 & (\bar{u}_j^\lambda \gamma^\mu (1 + \gamma^5) C_{\lambda\kappa} d_j^\kappa)] + \frac{ig}{2\sqrt{2}}W_\mu^- [(\bar{e}^\lambda \gamma^\mu (1 + \gamma^5) \nu^\lambda) + (\bar{d}_j^\kappa C_{\lambda\kappa}^\dagger \gamma^\mu (1 + \\
 & \gamma^5) u_j^\lambda)] + \frac{ig}{2\sqrt{2}}\frac{m_e^\lambda}{M}[-\phi^+(\bar{\nu}^\lambda (1 - \gamma^5) e^\lambda) + \phi^-(\bar{e}^\lambda (1 + \gamma^5) \nu^\lambda)] - \\
 & \frac{g}{2}\frac{m_e^\lambda}{M}[H(\bar{e}^\lambda e^\lambda) + i\phi^0(\bar{e}^\lambda \gamma^5 e^\lambda)] + \frac{ig}{2M\sqrt{2}}\phi^+ [-m_d^\lambda(\bar{u}_j^\lambda C_{\lambda\kappa}(1 - \gamma^5) d_j^\kappa) + \\
 & m_u^\lambda(\bar{u}_j^\lambda C_{\lambda\kappa}(1 + \gamma^5) d_j^\kappa)] + \frac{ig}{2M\sqrt{2}}\phi^- [m_d^\lambda(\bar{d}_j^\lambda C_{\lambda\kappa}^\dagger(1 + \gamma^5) u_j^\kappa) - m_u^\lambda(\bar{d}_j^\lambda C_{\lambda\kappa}^\dagger(1 - \\
 & \gamma^5) u_j^\kappa) - \frac{g}{2}\frac{m_u^\lambda}{M}H(\bar{u}_j^\lambda u_j^\lambda) - \frac{g}{2}\frac{m_d^\lambda}{M}H(\bar{d}_j^\lambda d_j^\lambda) + \frac{ig}{2}\frac{m_u^\lambda}{M}\phi^0(\bar{u}_j^\lambda \gamma^5 u_j^\lambda) - \\
 & \frac{ig}{2}\frac{m_d^\lambda}{M}\phi^0(\bar{d}_j^\lambda \gamma^5 d_j^\lambda)] + \bar{X}^+(\partial^2 - M^2)X^+ + \bar{X}^-(\partial^2 - M^2)X^- + \bar{X}^0(\partial^2 - \\
 & \frac{M^2}{c_w^2})X^0 + \bar{Y}\partial^2 Y + ig_{cw}W_\mu^+(\partial_\mu \bar{X}^0 X^- - \partial_\mu \bar{X}^+ X^0) + ig_{sw}W_\mu^-(\partial_\mu \bar{Y} X^- - \\
 & \partial_\mu \bar{X}^+ Y) + ig_{cw}W_\mu^-(\partial_\mu \bar{X}^- X^0 - \partial_\mu \bar{X}^0 X^+) + ig_{sw}W_\mu^-(\partial_\mu \bar{X}^- Y - \\
 & \partial_\mu \bar{Y} X^+) + ig_{cw}Z_\mu^0(\partial_\mu \bar{X}^+ X^+ - \partial_\mu \bar{X}^- X^-) + ig_{sw}A_\mu(\partial_\mu \bar{X}^+ X^+ - \\
 & \partial_\mu \bar{X}^- X^-) - \frac{1}{2}gM[\bar{X}^+ X^+ H + \bar{X}^- X^- H + \frac{1}{c_w^2}\bar{X}^0 X^0 H] + \\
 & \frac{1-2c_w^2}{2c_w}igM[\bar{X}^+ X^0 \phi^+ - \bar{X}^- X^0 \phi^-] + \frac{1}{2c_w}igM[\bar{X}^0 X^- \phi^+ - \bar{X}^0 X^+ \phi^-] + \\
 & igM_{sw}[\bar{X}^0 X^- \phi^+ - \bar{X}^0 X^+ \phi^-] + \frac{1}{2}igM[\bar{X}^+ X^+ \phi^0 - \bar{X}^- X^- \phi^0]
 \end{aligned}$$

$$S = -\frac{1}{16\pi G} \int \sqrt{-g}(R(g)+2\Lambda) d^4x$$



Modified Gravity



“Those that do not know geometry are not allowed to enter”.
Front Door of Plato’s Academy



Teleparallel Equivalent of General Relativity (TEGR)

In torsional formulation we use the vierbeins fields $\mathbf{e}_A(x^\mu)$ as dynamical variables, which at a manifold point x^μ form an orthonormal basis ($\mathbf{e}_A \cdot \mathbf{e}_B = \eta_{AB}$ with $\eta_{AB} = \text{diag}(1, -1, -1, -1)$).

In a coordinate basis they read as $\mathbf{e}_A = e_A^\mu \partial_\mu$ and the metric is given by

$$g_{\mu\nu}(x) = \eta_{AB} e_\mu^A(x) e_\nu^B(x),$$

with Greek and Latin indices used for the coordinate and tangent space respectively.

[Cai, Capozziello, De Laurentis, Saridakis, Rept.Prog.Phys. 79]

Teleparallel Equivalent of General Relativity (TEGR)

- Concerning the connection one introduces the Weitzenböck one, namely $\overset{\mathbf{w}}{\Gamma}_{\nu\mu}^{\lambda} \equiv e_A^{\lambda} \partial_{\mu} e_{\nu}^A$, and thus the corresponding torsion tensor becomes

$$T_{\mu\nu}^{\lambda} \equiv \overset{\mathbf{w}}{\Gamma}_{\nu\mu}^{\lambda} - \overset{\mathbf{w}}{\Gamma}_{\mu\nu}^{\lambda} = e_A^{\lambda} (\partial_{\mu} e_{\nu}^A - \partial_{\nu} e_{\mu}^A).$$

- The torsion tensor contains all information of the gravitational field, and its contraction provides the torsion scalar

$$T \equiv \frac{1}{4} T^{\rho\mu\nu} T_{\rho\mu\nu} + \frac{1}{2} T^{\rho\mu\nu} T_{\nu\mu\rho} - T_{\rho\mu}^{\rho} T^{\nu\mu}_{\nu},$$

which forms the Lagrangian of teleparallel gravity (in similar lines to the fact that the Ricci scalar forms the Lagrangian of general relativity).

$f(T)$ Gravity and $f(T)$ Cosmology

- One can use TEGR as the starting point of gravitational modifications. The simplest direction is to generalize T to a function $T + f(T)$ in the action:

$$S = \frac{1}{16\pi G} \int d^4x e [T + f(T) + L_m],$$

- Hence, we extract the Friedmann equations for $f(T)$ cosmology as

$$H^2 = \frac{8\pi G}{3}(\rho_m + \rho_r) - \frac{f}{6} + \frac{Tf_T}{3}$$
$$\dot{H} = -\frac{4\pi G(\rho_m + P_m + \rho_r + P_r)}{1 + f_T + 2Tf_{TT}},$$

Solving H0 and S8 tensions in f(T) Gravity

- We consider the following ansatz:

$$f(T) = -[T + 6H_0^2(1 - \Omega_{m0}) + F(T)], \quad (9)$$

where $F(T)$ describes the deviation from GR

The first Friedmann equation becomes

$$T(z) + 2\frac{F'(z)}{T'(z)}T(z) - F(z) = 6H_{\Lambda\text{CDM}}^2(z). \quad (10)$$

- In order to solve the H_0 tension, we need $T(0) = 6H_0^2 \simeq 6(H_0^{CC})^2$, with $H_0^{CC} = 74.03 \text{ km s}^{-1} \text{ Mpc}^{-1}$, while in the early era of $z \gtrsim 1100$ we require the Universe expansion to evolve as in ΛCDM , namely $H(z \gtrsim 1100) \simeq H_{\Lambda\text{CDM}}(z \gtrsim 1100)$. This implies $F(z)|_{z \gtrsim 1100} \simeq cT^{1/2}(z)$ (the value $c = 0$ corresponds to standard GR, while for $c \neq 0$ we obtain ΛCDM too).

Solving H0 and S8 tensions in f(T) Gravity

The effective gravitational coupling is given by

$$G_{\text{eff}} = \frac{G_N}{1 + F_T} . \quad (11)$$

Therefore, the perturbation equation becomes

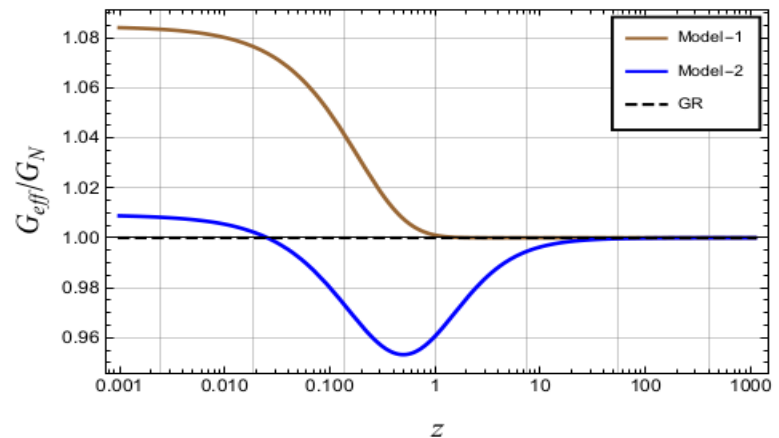
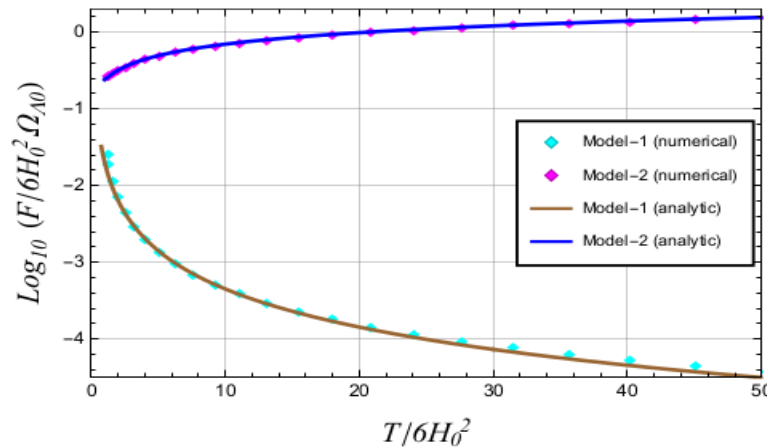
$$\delta'' + \left[\frac{T'(z)}{2T(z)} - \frac{1}{1+z} \right] \delta' = \frac{9H_0^2 \Omega_{m0}(1+z)}{[1 + F'(z)/T'(z)]T(z)} \delta . \quad (12)$$

Since around the last scattering moment $z \gtrsim 1100$ the Universe should be matter-dominated, we impose

$\delta'(z)|_{z \gtrsim 1100} \simeq -\frac{1}{1+z} \delta(z)$, while at late times we look for $\delta(z)$ that leads to an $f\sigma_8$ in agreement with redshift survey observations.

Solving H0 and S8 tensions in f(T) Gravity

By solving (10) and (12) with initial and boundary conditions at $z \sim 0$ and $z \sim 1100$, we can find the functional forms for the free functions of the $f(T)$ gravity that we consider, namely, $T(z)$ and $F(z)$, that can alleviate both H_0 and σ_8 tensions.

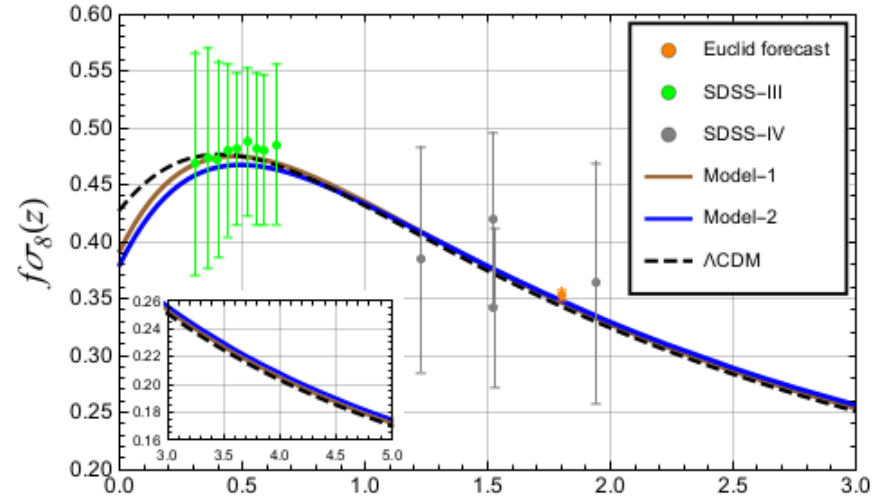
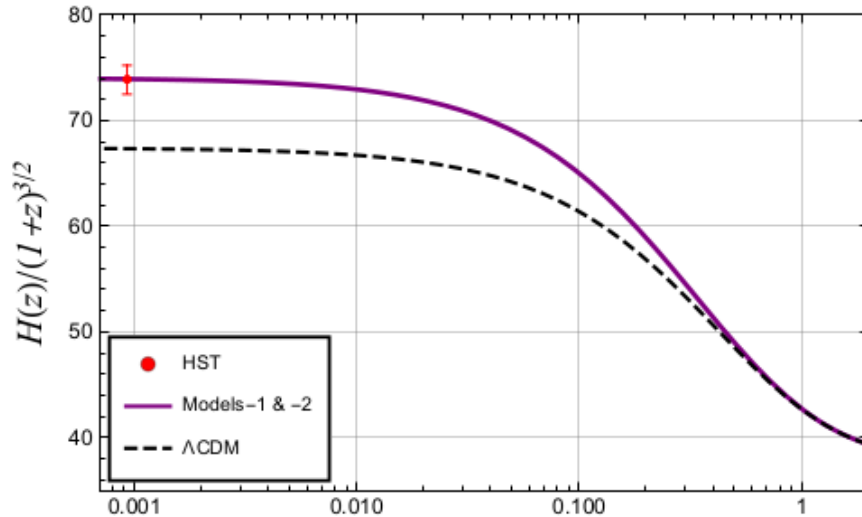


$$\text{Model-1: } F(T) \approx 375.47 \left(\frac{T}{6H_0^2} \right)^{-1.65}$$

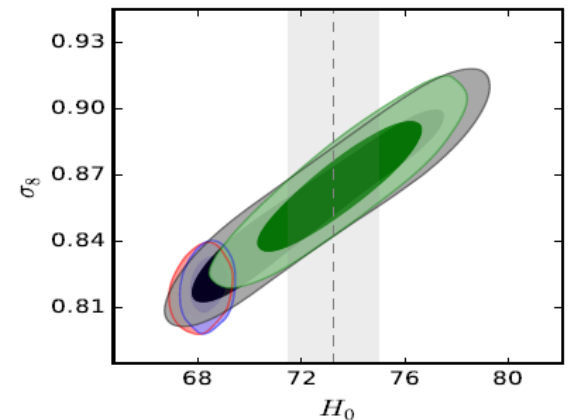
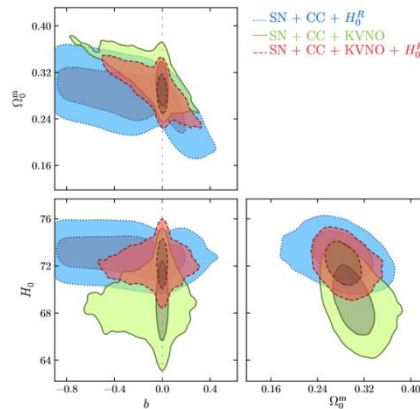
$$\text{Model-2: } F(T) \approx 375.47 \left(\frac{T}{6H_0^2} \right)^{-1.65} + 25 T^{1/2}.$$

[S-F Yan, P. Zhang, J_W Chen, X_Z Zhang, Y-F Cai, E.N. Saridakis, PRD 101]

Solving H0 and S8 tensions in f(T) Gravity



Parameter	CMB + BAO	CMB + BAO + H_0
$10^2 \omega_b$	$2.235^{+0.013}_{-0.013}$	$2.235^{+0.013}_{-0.013}$
ω_{cdm}	$0.1181^{+0.001}_{-0.001}$	$0.118^{+0.001}_{-0.001}$
$100\theta_s$	$1.041^{+0.00027}_{-0.00027}$	$1.041^{+0.00030}_{-0.00027}$
$\ln 10^{10} A_s$	$3.078^{+0.023}_{-0.023}$	$3.08^{+0.022}_{-0.022}$
n_s	$0.9678^{+0.0039}_{-0.0039}$	$0.9684^{+0.0039}_{-0.0044}$
τ_{reio}	$0.073^{+0.012}_{-0.013}$	$0.075^{+0.012}_{-0.012}$
n	$0.0043^{+0.0033}_{-0.0039}$	$0.0054^{+0.0020}_{-0.0020}$
$\log \alpha$	$10.00^{+0.081}_{-0.12}$	$10.03^{+0.06}_{-0.06}$
Ω_{F0}	$0.73^{+0.021}_{-0.028}$	$0.738^{+0.015}_{-0.015}$
H_0	$72.4^{+3.3}_{-4.1}$	$73.5^{+2.1}_{-2.1}$
σ_8	$0.855^{+0.023}_{-0.033}$	$0.866^{+0.02}_{-0.02}$
$\chi^2_{min}/2$	6480.48	6482.27

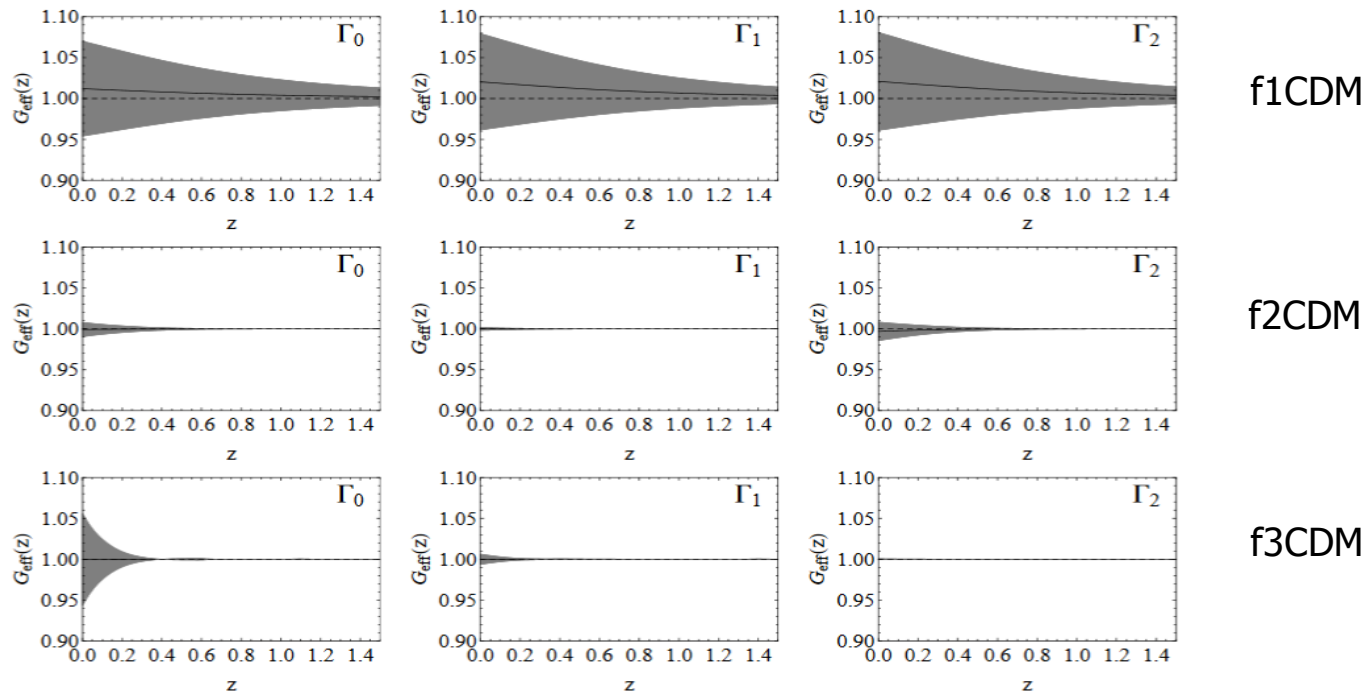


[S-F Yan, P. Zhang, J-W Chen, X_Z Zhang, Y-F Cai, E.N. Saridakis, PRD 101]

[J-W Chen, W. Luo, Y-F Cai, E.N. Saridakis, PRD 102]

[S. Basilakos, S. Nesseris, F. Anagnostopoulos, E.N. Saridakis, JCAP 2019]

Viable f(T) models



- In **f(T) gravity** we can indeed obtain $G_{\text{eff}}/G_{\text{N}} < 1$ for $z < 2$, without affecting the background evolution.
- **fσ8 tension** may be **alleviated**. [Nesseris, Basilakos, Saridakis, Perivolaropoulos, PRD 88]

In other modified gravities: Not possible

- This behavior **is not possible** in other **modified gravities**. e.g.:

$$S = \int d^4x \sqrt{-g} \left(\frac{1}{2} f(R, \phi, X) + \mathcal{L}_m \right) \quad X = -g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi$$

$$G_{\text{eff}}(a, k)/G_N = \frac{1}{F} \frac{f_{,X} + 4 \left(f_{,X} \frac{k^2}{a^2} \frac{F_{,R}}{F} + \frac{F_{,\phi}^2}{F} \right)}{f_{,X} + 3 \left(f_{,X} \frac{k^2}{a^2} \frac{F_{,R}}{F} + \frac{F_{,\phi}^2}{F} \right)} \quad F = F(R, \phi, X) = \partial_R f(R, \phi, X)$$

- $G_{\text{eff}}/G_N > 1$ for all models that **do not have ghosts** (i.e. with $f_R, f_{RR} > 0$).
- On the contrary, **f(T) gravity** has **second-order field equations** and moreover **perturbations are stable** in a large part of the parameter phase.

Solving H0 and S8 tensions in f(T) Gravity

- We conclude that the class of $f(T)$ gravity:
 $f(T) = -T - 2\Lambda/M_P^2 + \alpha T^\beta$, where only two out of the three parameters Λ , α , and β are independent (the third one is eliminated using Ω_{m0}), can alleviate both H_0 and σ_8 tensions with suitable parameter choices.
- Such kinds of models in $f(T)$ gravity could also be examined through galaxy-galaxy lensing effects [Z. Chen, W. Luo, Y.F. Cai and E.N. Saridakis, Phys.Rev.D 102 (2020) 10, 104044], strong lensing effects around black holes [S. Yan et. al, Phys.Rev.Res. 2 (2020) 2, 023164] and gravitational wave experiments [Y-F. Cai, C. Li, E.N. Saridakis and L. Xue, Phys. Rev. D 97, no. 10, 103513 (2018)].

Conclusions

- i) **Astrophysics** and **Cosmology** have become **precision** sciences.
- ii) A **huge amount of accumulating data** suggest possible **tensions** with **theoretical predictions** of Λ CDM paradigm.
- iii) **New Physics** or **paradigm shift** may be the **way out**
- iv) We can **modify** the **Universe content**, the **interactions**, or/and the **gravitational theory**.



Tensions in Cosmology



Workshop on Tensions in Cosmology

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THANK YOU!