Can the simplest generalizations of the null inertial mass density ( ) alleviate the H_0 tension?

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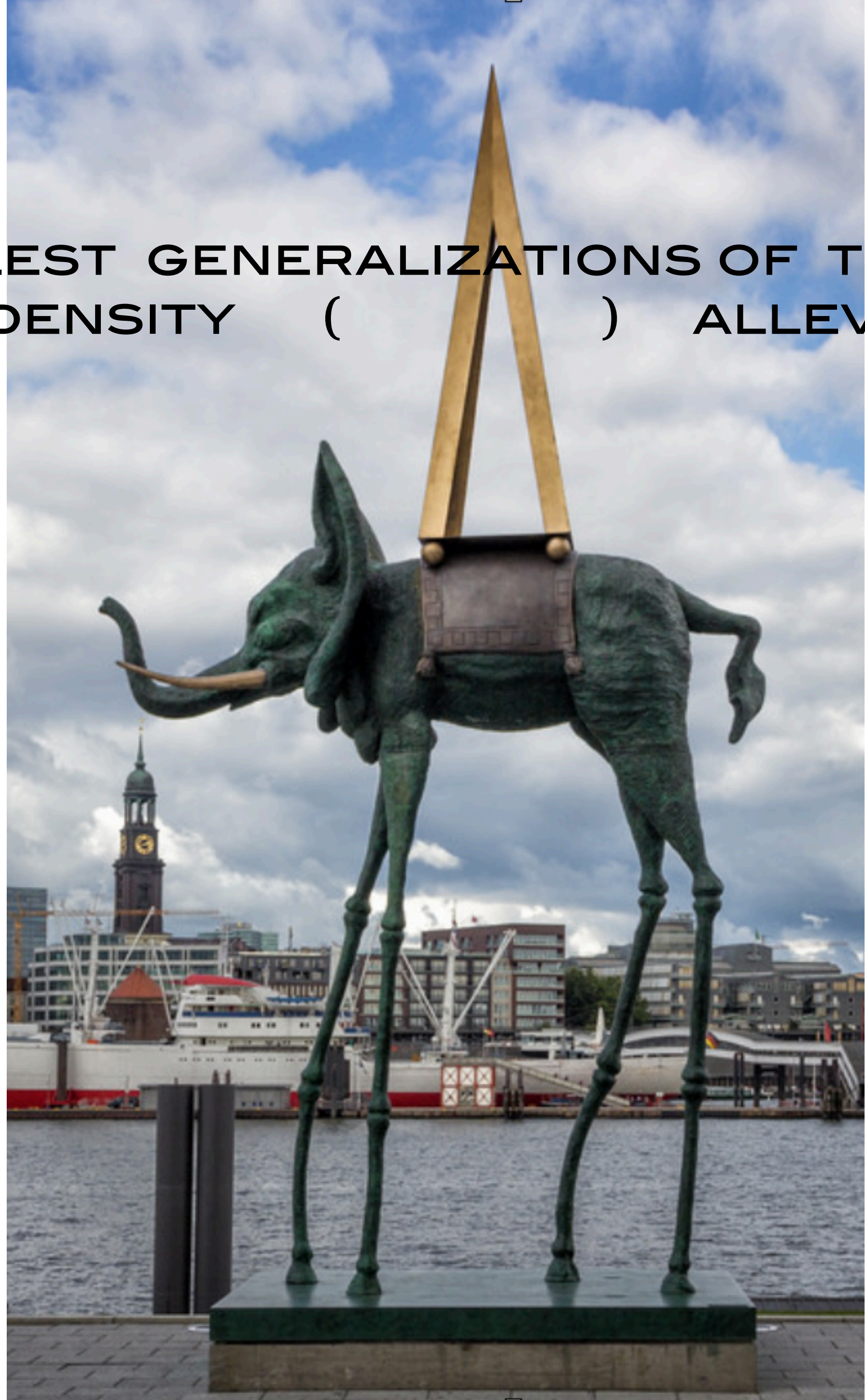

In collaboration with
Acquaviva, Akarsu
and Vazquez

PRD 104 023505,
2104.02623

Phys.Dark Univ. 38
101128 2203.01234

cost
EUROPEAN COOPERATION
IN SCIENCE & TECHNOLOGY

ANOMALIES & TENSIONS IN COSMOLOGY • COSMOVERSE •

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CosmoVerse@Lisbon Conference , Faculdade de Ciências da Universidade de Lisboa
30 May-1 June 2023

Legs of Λ *CDM*

General relativity - extrapolate Einstein's equations to scales above app.100 Mpc

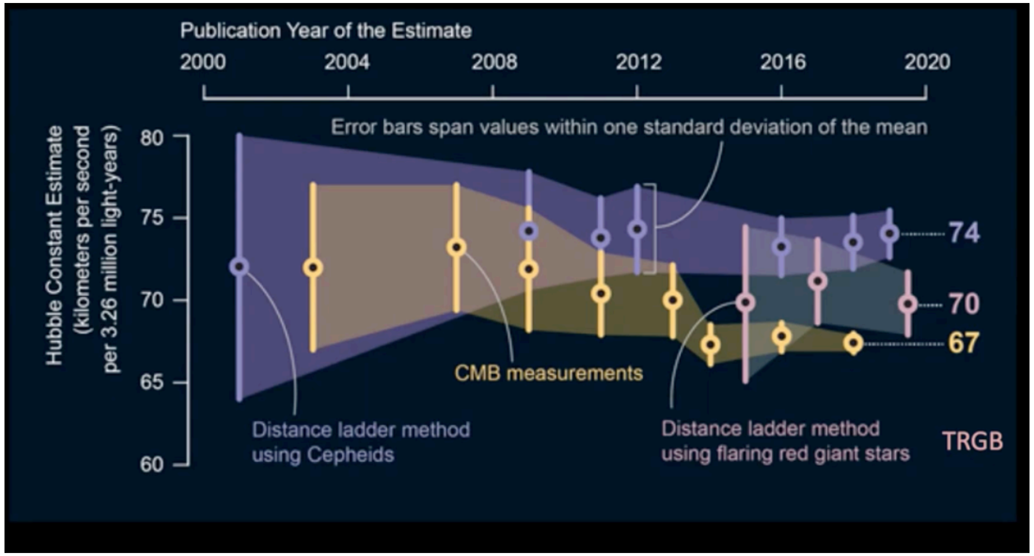
Matter content - Standard model of particle physics

General relativity

$$\nabla_\mu G^{\mu\nu} = 0 \to \nabla_\mu T^{\mu\nu} = 0.$$

Cosmological Principle - Universe's geometry and topology are as symmetric as possible

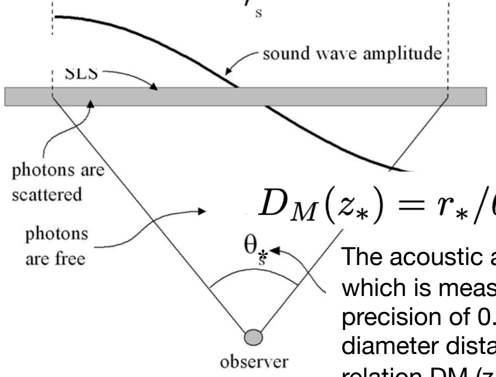
Hubble tension



- Early dark energy (EDE)
- Interacting dark energy (IDE) models,
- Phenomenologically Emergent Dark Energy
- Extra relativistic degrees of freedom at recombination, parametrized by Neff
- Sterile neutrinos, Goldstone bosons, axions, and neutrino asymmetry are typical examples to enhance the value of Neff
- Modified recombination and reionization histories through heating processes, variation of fundamental constants, or a non-standard CMB temperature-redshift relation
- Modified Gravity models
- Graduated dark energy models
- Decaying dark matter & interacting neutrinos
- a dynamical dark energy that assumes negative or vanishing density values at high redshifts

the co-moving sound horizon at CMB last scattering

$$r_* = \int_{z_*}^{\infty} c_s H^{-1}\,dz \qquad z_* \approx 1100$$



$$D_M(z_*) = r_*/\theta_*$$

the pre-recombination Universe

$$D_M(z_*) = c\int_0^{z_*} H^{-1}\,dz$$

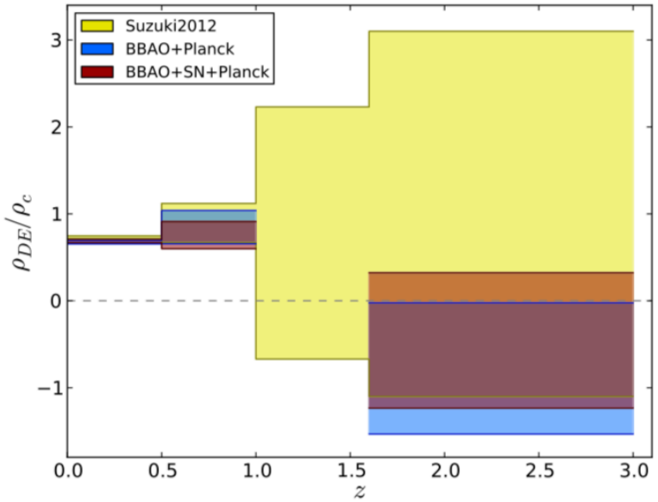
KNOX, MILLEA, PHYS. REV. D 101, 043533 2020, 1908.03663.

The acoustic angular scale on the sky, θ∗, which is measured almost model independently with a precision of 0.03% determines the comoving angular diameter distance to last scattering DM (z∗) through the relation DM (z∗) = r∗/θ∗.

AUBOURG *ET AL.* (BOSS COLLAB.) PRD 92, 123516, 1411.1074

SAHNI, SHAFIELOO, STAROBINSKY PRD 92, 123516, 1406.2209

COSMOLOGY INTERTWINED , J. HIGH EN. ASTROPHYS. 2204, 002 (2022)



- DE energy density that attains negative values at high redshifts can enhance H(z) at low redshifts, H0 even further.

AKARSU ET.AL. , EUR. PHYS. J. C 80 (2020) 1050, 2004.04074

Model-independent reconstruction of the Interacting Dark Energy Kernel —〉 a sign change in the direction of the energy transfer between DE and DM

ESCAMILLA ET.AL. 2305.16290

Inertial mass density $\varrho = \rho + p$

$$\Theta = D^{\mu} u_{\mu}$$

$$\nabla_{\nu} u_{\mu} = D_{\nu} u_{\mu} - \dot{u}_{\mu} u_{\nu}$$

$$D_{\nu} u_{\mu} = \frac{1}{3} \Theta h_{\mu\nu}$$

$$u_{\mu} u^{\mu} = -1$$

$$\nabla_{\nu} u^{\mu} u_{\mu} = 0$$

The EMT can be decomposed relative to u_{μ}, in the form

$$T_{\mu\nu} = (\rho + p) u_{\mu} u_{\nu} + p g_{\mu\nu}$$

Einstein field equations arises from the twice contracted Bianchi Identity implying

$$\nabla_{\mu} G^{\mu\nu} = 0 \rightarrow \nabla_{\mu} T^{\mu\nu} = 0$$

Projecting parallel and orthogonal to u_{μ}, we obtain energy and momentum conservation equations,

$$\dot{\rho} + \Theta \varrho = 0 \qquad D^{\mu} p + (\rho + p) \dot{u}^{\mu} = 0 \qquad \varrho = \rho + p$$

BARROW, PLB 235 (1990)

$$\varrho = \gamma \rho_0 \left(\frac{\rho}{\rho_0} \right)^{\lambda}$$

Λ null inertial mass density $\gamma = 0$ What will the data say? At which scales?

ACQUAVIVA, AKARSU, KATIRCI, VAZQUEZ, PRD 104 023505 2021 2104.02623

constant deviation from null inertial mass density $\lambda = 0$

BOUHMADI-LOPEZ ET. AL., IJMPD 24 1550078 (2015) 1407.2446

dynamical deviation from null inertial mass density

non-trivial behaviors

AKARSU,BARROW,ESCAMILLA,VAZQUEZ, PRD 101 063528 1912.08751

$$\rho + p = (1 + w) \rho_0 (1 + z)^{3(1+w)} \quad \text{for } w\text{CDM}$$

Graduated dark energy -a spontaneous sign switch in Λ

AKARSU, BARROW , ESCAMILLA, VAZQUEZ, PRD , 101 063528 1912.08751

$$\varrho \propto \rho^{\lambda} < 0 \quad \text{with} \quad \lambda < 1$$

its energy density ρ dynamically takes negative values in the finite past.

For large negative values of λ , it creates a phenomenological model described by a smooth function that approximately describes the Λ spontaneously switching sign in the late universe to become positive today.

the latest combined observational data sets of PLK+BAO+SN+H

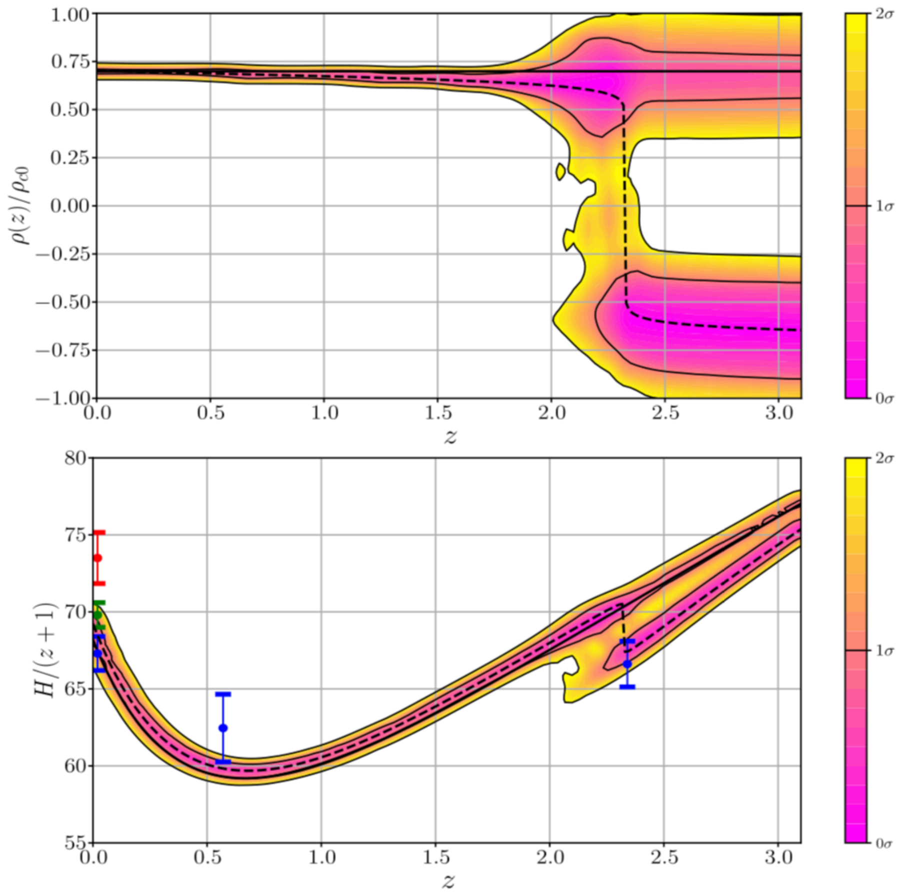


FIG. 7: Top panel: $\rho_{\rm gDE}/\rho_{\rm c0}$ versus redshift z for $\lambda = -20$ displays the maximum predicted that $\rho_{\rm gDE}$ changes sign at $z \sim 2.3$. Bottom: $H(z)/(1+z)$ function. Include the latest BAO data points [37] (blue bars) where $H_0 = 67.3 \pm 1.1$, the Planck 2018 [9] $H_0 = 67.4 \pm 0.5$ data (red bar) and the TGRB model independent [22] $H_0 = 69.8 \pm 0.8$ data (green bar). Black dashed line corresponds to best-fit values of gDE and solid black line corresponds to LCDM. We note that, due to the jump at $z \sim 2.3$, the gDE model is not in tension with the BAO Ly-α data from $z = 2.34$ in contrast to ΛCDM model and also gDE gives larger H_0 values w.r.t. ΛCDM model and thereby relaxes H_0 tension.

ΛsCDM model, AKARSU,KUMAR,ÖZÜLKER, VAZQUEZ, 2108.09239

$$\frac{H^2}{H_0^2} = \Omega_{\rm r0}(1+z)^4 + \Omega_{\rm m0}(1+z)^3 + \Omega_{\Lambda_{\rm s}0}\,{\rm sgn}[z_\dagger - z].$$

Two simplest **ΛCDM** extensions : Simple graduated DE or curvature

Can these, together or separately, successfully realize such a scenario?

$$\frac{H^2}{H_0^2} = \Omega_{\rm ci0}\left[1+3(1+w_{\rm ci0})\ln(1+z)\right] + \Omega_{k0}(1+z)^2 + \Omega_{\rm m0}(1+z)^3 + \Omega_{\rm r0}(1+z)^4,$$

Simple graduated DE

$\varrho < 0$

promotes null inertial mass density of conventional vacuum energy to an arbitrary constant.

$w_{\rm ci0} < -1\,,\ \rho_{\rm ci0} > 0$

Reminiscent of PEDE, decreasing with increasing z, yet no extra dof

The spatial curvature, in the case of spatially closed Universe

$\Omega_{k0} < 0 \qquad w = -1/3$

DI VALENTINO, MELCHIORRI, SILK, NATURE ASTRON. 1911.02087, 2003.04935 , HANDLEY, 1908.09139

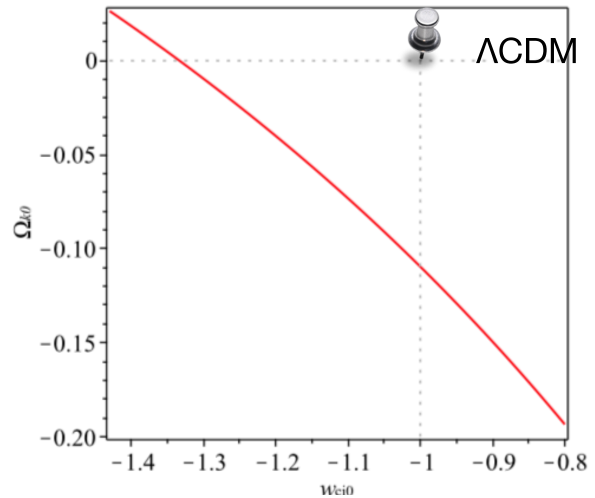
The fact that the Planck data favor positive spatial curvature on top of the ΛCDM model implying such dark energy models

the de Sitter future of the ΛCDM

$\dot{H} = 0$

$\dot{H} = -\frac{1}{2}\varrho \neq 0$

BOUHMADI-LOPEZ ET.AL., IJMPD 24 1550078 (2015) 1407.2446



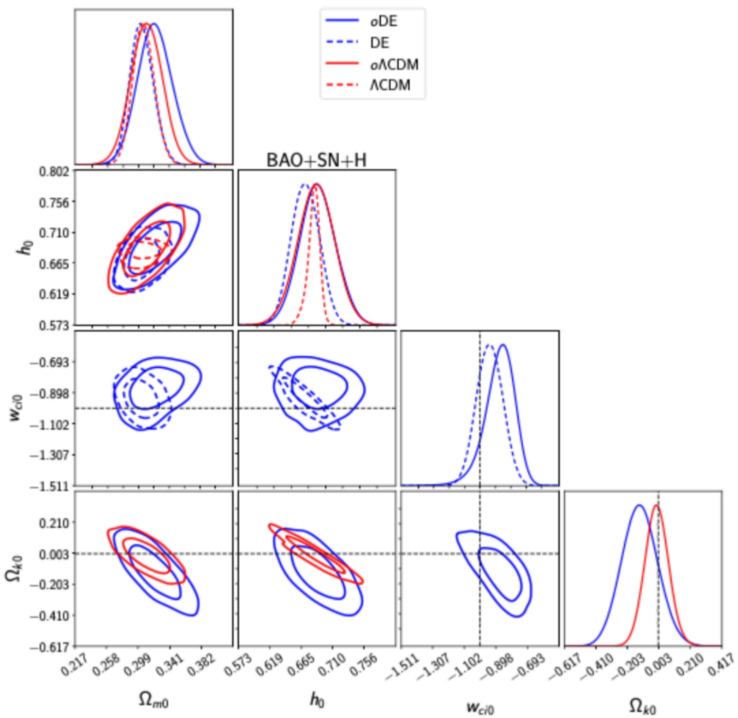
it resembles Λ today,
alas leading to a future singularity dubbed as the Little Sibling of the Big Rip (LSBR)

$\varrho < 0 \qquad w_{\rm ci0} < -1\,,\ \rho_{\rm ci0} > 0$

a finite future bounce $\quad \varrho > 0 \quad w_{\rm ci0} > -1\,,\ \rho_{\rm ci0} > 0$

Does it compensate to make flat again?

Observational analysis - (BAO+SN+H)



Dataset	BAO+SN+H			
	ΛCDM	oΛCDM	DE	oDE
Ω_{m0}	0.307 ± 0.014	0.310 ± 0.020	0.304 ± 0.015	0.322 ± 0.022
$\Omega_{b0}h_0^2$	0.02204 ± 0.00047	0.02204 ± 0.00046	0.02204 ± 0.00047	0.02204 ± 0.00045
h_0	0.6827 ± 0.0088	0.6862 ± 0.0268	0.6706 ± 0.0202	0.6884 ± 0.0260
w_{ci0}	-1	-1	-0.937 ± 0.084	-0.872 ± 0.097
Ω_{k0}	—	-0.011 ± 0.077	—	-0.122 ± 0.117
$\varrho_{ci} \times 10^{31}$ [g cm^{-3}]	0	0	3.46 ± 4.76	7.65 ± 5.72
Ω_{ci0}	0.693 ± 0.014	0.700 ± 0.064	0.696 ± 0.015	0.800 ± 0.101
Ω_{kci0}	—	0.690 ± 0.020	—	0.678 ± 0.022
z_{ci*}	—	—	< -0.96 or $\gtrsim 10^7$	< -0.78
z_{kci*} (z_{kcc*})	—	> 1.26	—	> 0.92
$-2\ln\mathcal{L}_{max}$	58.97	58.96	58.28	56.91
$\ln\mathcal{Z}$	-36.54 ± 0.19	-38.38 ± 0.21	-37.96 ± 0.21	-38.00 ± 0.21
$\Delta\ln\mathcal{Z}$	0	-1.84 ± 0.28	-1.42 ± 0.28	-1.46 ± 0.28

- ❑ The oDE model, having the lowest $-2 \ln L_{max}$ value, but the Bayesian evidence on the other hand suggests that there is a significant evidence for preferring the ΛCDM model over the extended models, as for which $|\Delta \ln Z| \sim 1.5$.
- ❑ Contrary to our initial expectations, the simple-gDE worsens the so-called H_0 tension. The reason is being that the data favor $\varrho_{ci} = (3.46 \pm 4.76) \times 10^{-31}$ g cm^{-3} ($w_{ci0} = -0.937 \pm 0.084$) rather than a definitely negative inertial mass destiny.

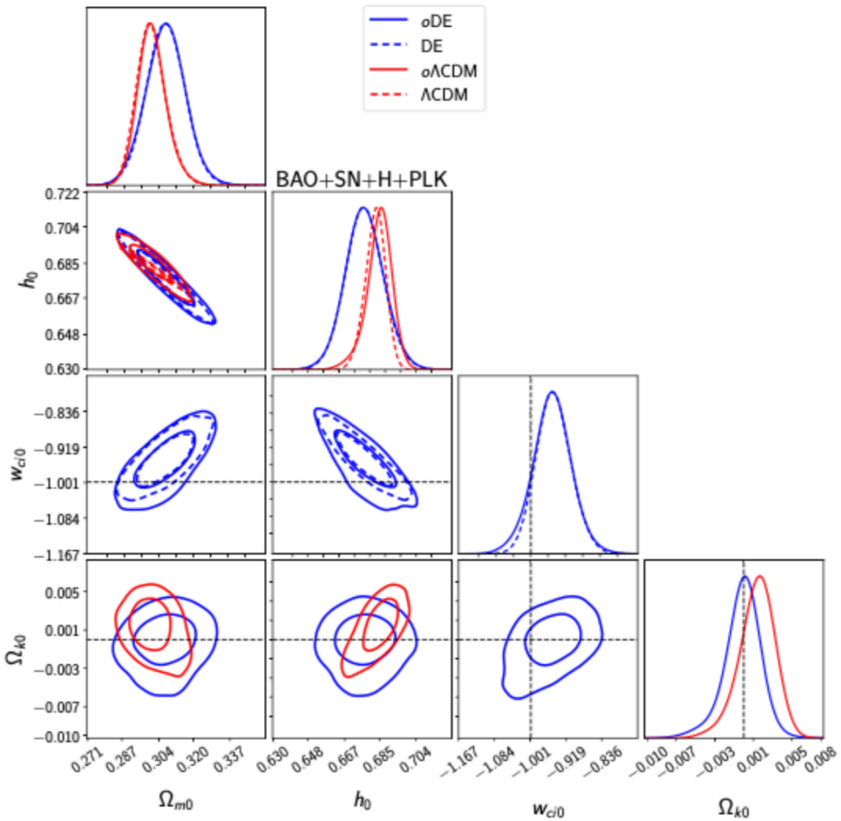
- ☑ There is no evidence to prefer the oΛCDM model, which yields $\Omega_{k0} = -0.011 \pm 0.077$ consistent with spatially flat Universe, over the oDE model, which yields $\Omega_{k0} = -0.122 \pm 0.117$ suggesting spatially closed Universe with high significance.
- ☑ So, the inclusion of spatial curvature however lifts H_0 to the values larger than those allowed within the ΛCDM model with
- ☑ the negative correlation between Ω_{k0} and w_{ci0}.

In both models, this happens because of the closed space ($\Omega_{k0} < 0$), whereas the simple-gDE opposes it—notice that the energy density of the simple-gDE never crosses below zero in the past, but in the far future ($z_{ci*} < -0.78$).

Simple MC code [1411.1074]

https://github.com/slosar/april, version May 2019.

Observational analysis - (BAO+SN+H+PLK)



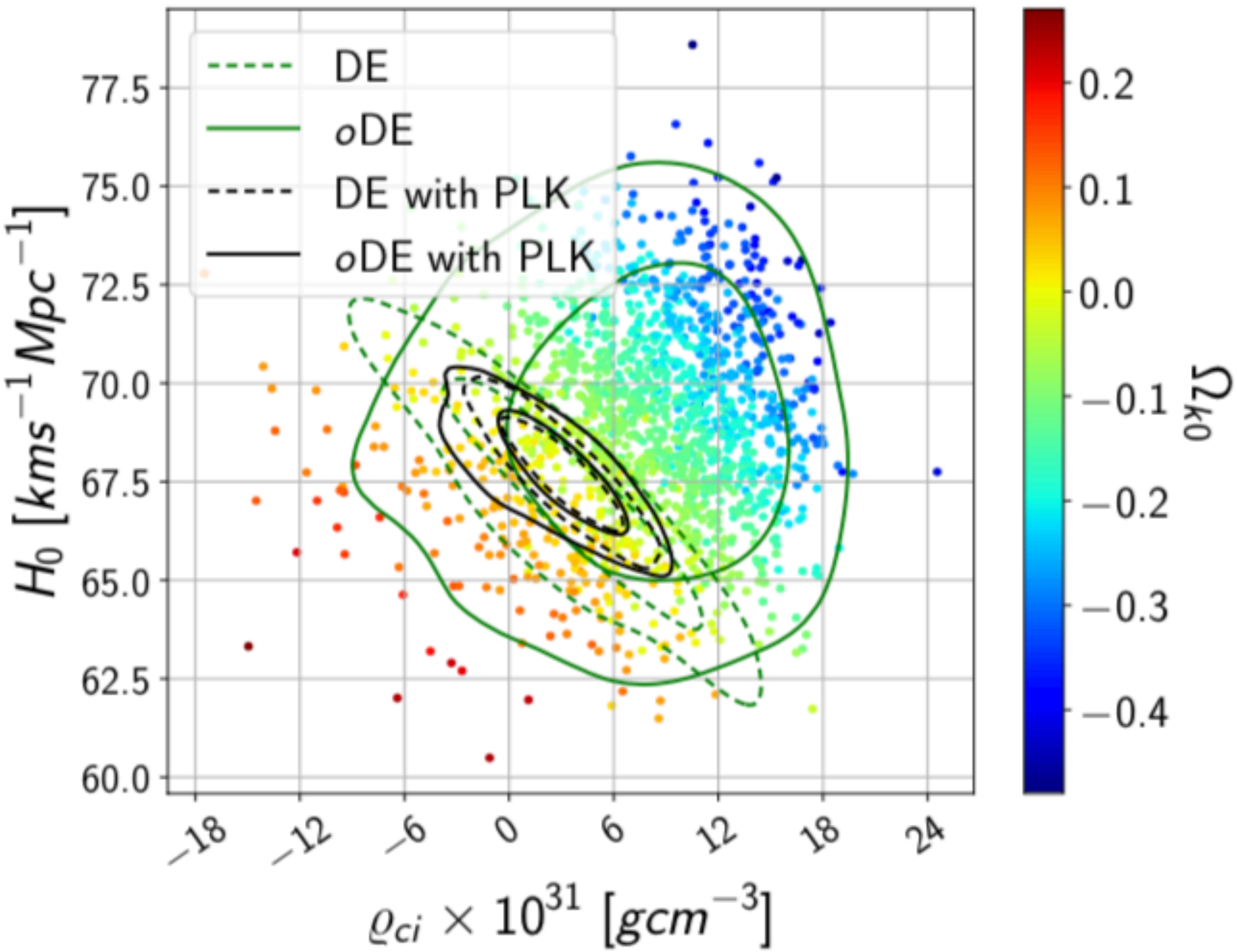
Dataset	BAO+SN+H+PLK			
	ΛCDM	oΛCDM	DE	oDE
Ω_{m0}	0.3005 ± 0.0068	0.3009 ± 0.0067	0.3070 ± 0.0088	0.3071 ± 0.0091
$\Omega_{b0}h_0^2$	0.02245 ± 0.00015	0.02237 ± 0.00017	0.02242 ± 0.00015	0.02241 ± 0.00017
h_0	0.6829 ± 0.0052	0.6849 ± 0.0067	0.6772 ± 0.0097	0.6773 ± 0.0099
w_{ci0}	-1	-1	-0.948 ± 0.041	-0.951 ± 0.045
Ω_{k0}	—	0.0012 ± 0.0018	—	-0.0001 ± 0.0019
$\varrho_{ci} \times 10^{31}$ [g cm^{-3}]	0	0	3.06 ± 2.28	2.85 ± 2.58
Ω_{ci0}	0.6994 ± 0.0068	0.6977 ± 0.0065	0.6929 ± 0.0088	0.6929 ± 0.0095
Ω_{kci0}	—	0.6991 ± 0.0067	—	0.6928 ± 0.0091
z_{ci*}	—	—	< -0.99	< -0.99
z_{kci*} (z_{kcc*})	—	> 9.62	—	> 6.64
$-2\ln\mathcal{L}_{\max}$	60.46	59.27	58.24	58.24
$\ln\mathcal{Z}$	-42.02 ± 0.26	-43.78 ± 0.26	-42.19 ± 0.25	-44.13 ± 0.27
$\Delta\ln\mathcal{Z}$	0	-1.76 ± 0.37	-0.17 ± 0.36	-2.11 ± 0.37

The joint data set, including the Planck data, presents no evidence for a deviation from spatial flatness, but almost **the same evidence** for a cosmological constant and the simple-gDE with an inertial mass density of order O(10^{-12})eV^4.

Simple MC code [1411.1074]

https://github.com/slosar/april, version May 2019.

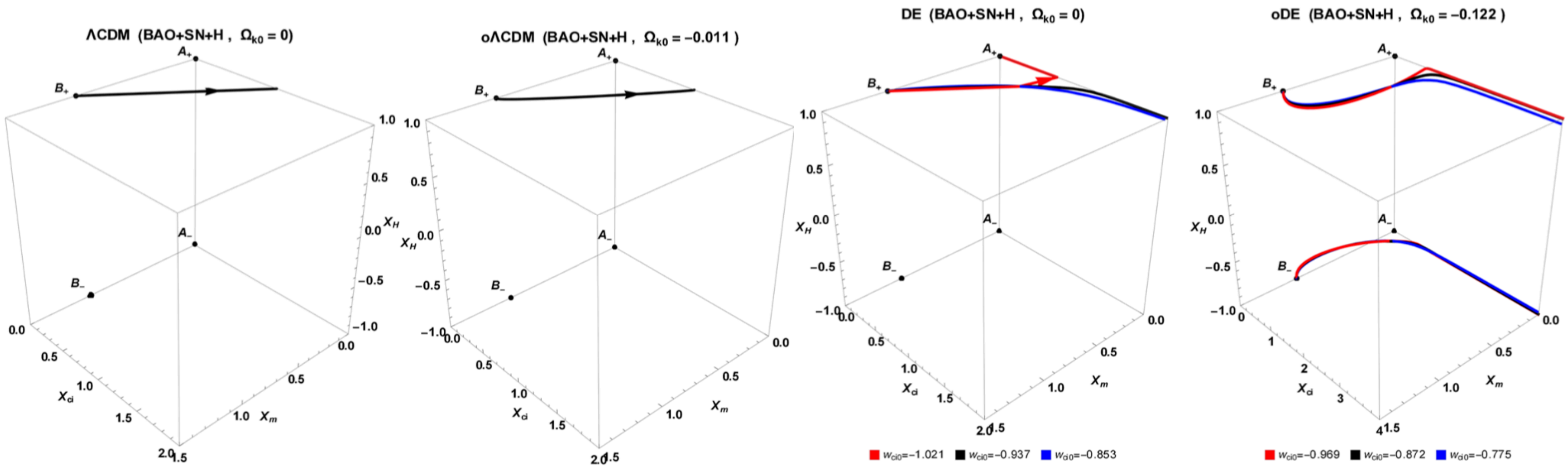
Interplay between H_0 , ϱ and Ω_{k0}



The joint data set, including the Planck data, presents no evidence for a deviation from spatial flatness, but almost **the same evidence** for a cosmological constant and the simple-gDE with an inertial mass density of order $O(10^{-12})eV^4$.

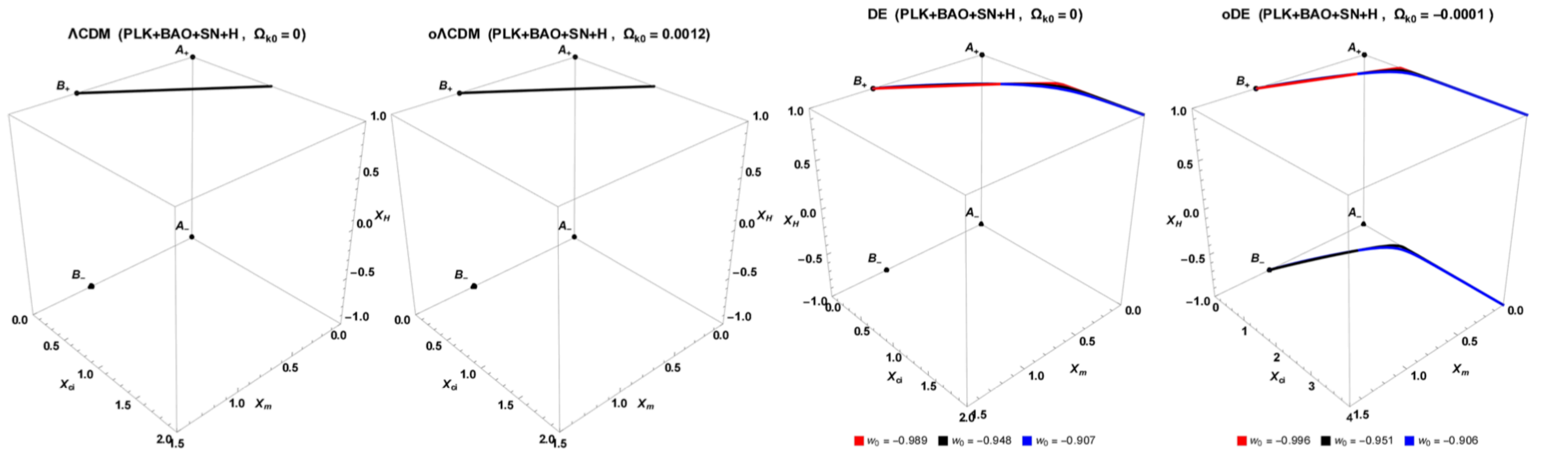
Vacuum inertial mass density may be a constant of nature, rather than vacuum energy density

Dynamical analysis - asymptotic behaviour of the models - (BAO+SN+H)



- Two distinct futures depending on the sign of inertial mass density, rather than de Sitter future of ΛCDM model.
- For spatially flat simple gDE case (DE) constrained without PLK allows $\varrho < 0$.

Dynamical analysis - asymptotic behaviour of the models (BAO+SN+H+PLK)



- Recollapsing of the Universe in finite future is a generic behavior of simple gDE models as $\varrho > 0$ within 68% CL independent of whether the PLK data is included or not.

Suggestions to address this tension by reanalyzing the cosmological data by breaking down of the RW framework

e.g., allowing anisotropic expansion in the late universe; suggesting [Colin 2017, 2019, Secrest:2020has,Krishnan 2021,Luongo 2021]

Anisotropic Hubble Expansion in Pantheon+ Supernovae , arXiv:2304.02718, they are saying that H_0 is larger in a hemisphere encompassing the CMB dipole direction. They are looking for dipole, what happens for quadrupole?

K:MIGKAS et al. Astron. Astrophys. 2004.03305
AKRAMI [PLANCK COLL.] A&A 641, A7 (2020), 2212.13569
WILCZYNSKA et. Al. Sci.Adv. 6 (2020) 17, 2003.07627

Zwicky Transition Facility SNe Ia sample test the isotropy of the expansion rate, i.e. the Hubble constant H0, in the nearby Universe and it shows some indications for potential deviations from isotropy and forecasts suggest the exciting possibility to strongly confirm or refute this claim.

Potential signature of a quadrupolar Hubble expansion in Pantheon+ supernovae,
COWELL, DHAWAN, MACPHERSON, 2212.13569

Scalar field emulator via deformed vacuum energy: Application to dark energy

Deformed vacuum energy [Akarsu,Katırcı,Sen,Vazquez, 2004.14863]
generalization of the usual VE: by allowing anisotropic pressure whilst preserving zero inertial mass density on average

$$T_{\mu\nu} = \rho u_\mu u_\nu + p_{\rm iso} h_{\mu\nu} + \pi_{\mu\nu}$$

trace-free anisotropic pressure

$$\nabla_\mu G^{\mu\nu} = 0 \to \nabla_\mu T^{\mu\nu} = 0.$$

$$\dot{\rho} + \Theta\varrho + \sigma^{\mu\nu}\pi_{\mu\nu} = 0$$

$$\mathrm{D}^\mu p_{\rm iso} + (\varrho + \pi^\mu_\mu)\dot{u}^\mu + (\mathrm{div}\pi)^\mu = 0,$$

$$\Theta = D^\mu u_\mu$$

$$\pi_{\mu\nu} = T_{\langle\mu\nu\rangle}$$

$$\sigma_{\mu\nu} = D_{\langle\mu}u_{\nu\rangle}$$

$$\dot{u}_\mu = u_\nu \nabla^\nu u_\mu$$

$$\nabla_\nu u_\mu = D_\nu u_\mu - \dot{u}_\mu u_\nu$$

$$D_\nu u_\mu = \frac{1}{3}\Theta h_{\mu\nu} + \sigma_{\mu\nu}$$

the most general form of the EMT, accommodated by LRS Bianchi type-I metric

modified theories may contribute as like anisotropic source.
A comment: Faraoni&Cote (2018) 1808.02427, Akarsu et al. (2020) 1903.06679

GR with **anisotropy** + **a fluid still has null imd**

GEOMETRY : LRS Bianchi type-I metric described by the line element

$$ds^2 = -dt^2 + S^2\left[e^{\frac{4}{\sqrt{6}}\varphi}dx^2 + e^{-\frac{2}{\sqrt{6}}\varphi}(dy^2 + dz^2)\right]$$

shear scalar is squared of the time derivative of spatial metric.

$$\pi_2^2 - \pi_1^1 = \gamma$$

$$\sigma^2 = \sigma_{\mu\nu}\sigma^{\mu\nu} = \dot{\varphi}^2$$

MATTER : anisotropic extension of vacuum energy

$$\varrho_x = \rho + p_{\rm iso} + \pi_1^1$$

$$\varrho_y = \varrho_z = \rho + p_{\rm iso} + \pi_2^2$$

$$\varrho = \frac{1}{3}\left[3\rho + 3p_x + 2\gamma\rho\right]$$

$$\pi_2^2 - \pi_1^1 = \gamma\rho$$

$$p_{\rm y(z)} = p_{\rm x} + \gamma\rho$$

$$H = \frac{\dot{S}}{S} = \frac{H_x + 2H_y}{3} \quad \text{and} \quad \sigma^2 = \frac{3}{2}(H_x - H)^2.$$

$= 0$?

$$w_x = -1 - \frac{2\gamma}{3}$$

particular relation with EoS parameter and skewness

$$T_\mu{}^\nu = \mathrm{diag}\left[-1, -1 - \frac{2}{3}\gamma, -1 + \frac{1}{3}\gamma, -1 + \frac{1}{3}\gamma\right]\rho,$$

cosmic triad and arbitrary number of this EMT oriented in arbitrary directions on average, would also lead, stochastically, to conventional vacuum energy,

No correspondence from known anisotropic sources

(i.e. vector fields, topological defects)

scalar (canonical) field emulator

Einstein field equations + LRS Bianchi-I + anisotropically deformed vacuum energy

$$3\mathcal{H}^2 = \frac{1}{2}\sigma^2 + \rho_{\mathrm{dv}},$$

$$-2\dot{\mathcal{H}} - 3\mathcal{H}^2 = \frac{1}{2}\sigma^2 - \rho_{\mathrm{dv}},$$

$$\dot{\sigma} + 3\mathcal{H}\sigma = -\sqrt{\frac{2}{3}}\gamma\rho_{\mathrm{dv}},$$

direct observable

shear propagation equation

Einstein field equations + RW metric + canonical scalar fields

$$3H^2 = \frac{1}{2}\dot{\phi}^2 + V(\phi),$$

$$-2\dot{H} - 3H^2 = \frac{1}{2}\dot{\phi}^2 - V(\phi),$$

$$\ddot{\phi} + 3H\dot{\phi} = -\frac{\mathrm{d}V}{\mathrm{d}\phi}$$

Klein Gordon equation

under the following transformations

$$\mathcal{H} \to H$$

$$\sigma \to \dot{\phi}$$

$$\rho_{\mathrm{dv}} \to V(\phi)$$

$$\gamma \to \sqrt{\frac{3}{2}}\frac{1}{V}\frac{\mathrm{d}V}{\mathrm{d}\phi}$$

We can reconsider the cosmologies employing a canonical SF.

the deformed vacuum energy + the shear scalar

Defining the effective quantities,

$$w_{\rm eff} = \frac{p_{\rm eff}}{\rho_{\rm eff}} = \frac{\sigma^2/2 - \rho_{\rm dv}}{\sigma^2/2 + \rho_{\rm dv}}$$

Shear propagation equation -> continuity equation for the effective source defined from the cooperation of the deformed vacuum with the shear scalar

$$\dot{\rho}_{\rm eff} + 3\mathcal{H}\rho_{\rm eff}(1 + w_{\rm eff}) = 0,$$

the non-negativity condition on the density of the deformed vacuum energy- along with that the shear scalar is non-negative definite guarantee that

$$w_{\rm eff} < -\frac{1}{3} \qquad -1 \leq w_{\rm eff} \leq 1$$

$$\sigma^2 < \rho_{\rm dv}$$

the role of the flatness of the potential is taken over by the ratio-squared of the rate of change of the energy density of the deformed vacuum to the shear scalar

$$\epsilon \to \frac{\gamma^2}{3} = \frac{1}{2}\frac{\dot{\rho}_{\rm dv}^2}{\rho_{\rm dv}^2}\frac{1}{\sigma^2}.$$

you can construct the anisotropic counterpart cosmologies + a bonus

Canonical SF's

$$w_\phi = \frac{p_\phi}{\rho_\phi} = \frac{\dot{\phi}^2/2 - V}{\dot{\phi}^2/2 + V}$$

KG-> continuity eq. for the SF

$$\dot{\rho_\phi} + 3\mathcal{H}\rho_\phi(1 + w_\phi) = 0$$

no-go theorem forbids a single canonical SF with a non-negative potential to cross below the w=-1 boundary of the usual vacuum energy, viz., its EoS parameter is confined to the range

$$w_\phi < -\frac{1}{3} \qquad -1 \leq w_\phi \leq 1$$

$$\dot{\phi}^2 < V$$

slow roll parameter for the SF

$$\dot{\phi}^2 << V \qquad \epsilon = \frac{1}{2}(\frac{1}{V}\frac{{\rm d}V}{{\rm d}\phi})^2$$

what you can construct cosmologically with SF,

A simplest example:

COSMOLOGY DE's by SF which slowly rolls down –epitome of quintessence models

WHAT IS THE SHEAR SCALAR -DEFORMED VE ALLIANCE?

$$\dot{\rho}_{\rm eff} + 3\mathcal{H}\rho_{\rm eff}(1+w_{\rm eff}) = 0, \qquad \rho_{\rm eff} = \rho_{\rm eff0}\, e^{3\int(1+w_{\rm eff})\, d\ln(1+z)}.$$

$$3\mathcal{H}^2 = \sum_i \rho_{i0}(1+z)^{3(1+w_i)} + \rho_{\rm eff},$$

$$\rho_{\sigma^2} \equiv \frac{\sigma^2}{2} = \frac{1+w_{\rm eff}}{2}\rho_{\rm eff}, \qquad \rho_{\rm dv} = \frac{1-w_{\rm eff}}{2}\rho_{\rm eff}.$$

drastic deviation from stiff fluid character of shear scalar

	$\Lambda\mathrm{CDM}_{\sigma^2}$	$\mathrm{dv}w\mathrm{CDM}_{\sigma^2}$
$\rho_{\rm eff}$	$\rho_{\rm dv} + \rho_{\sigma^2 0}(1+z)^6$	$\rho_{\rm eff0}(1+z)^{3(1+w_{\rm eff})}$
$w_{\rm eff}$	$\frac{\rho_{\sigma^2 0}(1+z)^6 - \rho_{\rm dv}}{\rho_{\sigma^2 0}(1+z)^6 + \rho_{\rm dv}}$	const. ≥ -1
ρ_{σ^2}	$\rho_{\sigma^2 0}(1+z)^6$	$\frac{1}{2}(1+w_{\rm eff})\rho_{\rm eff0}(1+z)^{3(1+w_{\rm eff})}$
$\rho_{\rm dv}$	const	$\frac{1}{2}(1-w_{\rm eff})\rho_{\rm eff0}(1+z)^{3(1+w_{\rm eff})}$
γ	0	$-3\sqrt{\frac{1+w_{\rm eff}}{2}\left[1+\frac{\rho_{\rm m0}}{\rho_{\rm eff0}}(1+z)^{-3w_{\rm eff}}\right]}$

see Akarsu et al. 1905.06949 for a anisotropic generalization of LCDM

see for some deviations :

Madsen (1988) Pimentel (1989), Faraoni&Cote (2018) 1808.02427, Akarsu et al. (2020) 1903.06679

Conclusions

- Discussion on the possible alleviation of H_0 tension with dark energy models with negative energy density values in the past.
- Two minimal ways to achieve, let the curvature to vary

  let the promotion of null inertial mass density of Λ, (simple gDE)

  together or separately.
- **We confirmed that fixing spatial flatness with assumption hidden possible deviations from ΛCDM.**
- **Significant deviation from spatial flatness along with a simple-gDE of a positive inertial mass density which in opposition to each other imply no robust improvement in the H_0 tension.**
- **There is the same evidence for the ΛCDM model and the DE model (simple-gDE) with a positive inertial mass density at the order of O (10^{-12}) eV^4, namely, $\rho_{ci} = (3.06 \pm 2.28) \times 10^{-31}$ g/cm^3.**
- **Even the null inertial mass density is ruled out and $\rho > 0$ within 68% CL.**
- Energy-momentum squared gravity generates logarithmic correction keeping constant inertial mass density for each standard source with w,

AKARSU, BARROW &UZUN, EUR. PHYS. J. C 79 (2019) 846, 1903.11519.

for a comparative detailed analysis: ACQUAVIVA &KATIRCI, PDU 38 101128 2203.01234.

- **Can it be derived/predicted from a fundamental theory of physics?**
- **We can construct anisotropic counterpart of scalar field cosmologies.**

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