Pseudomagic quantum states: when physics meets computer science

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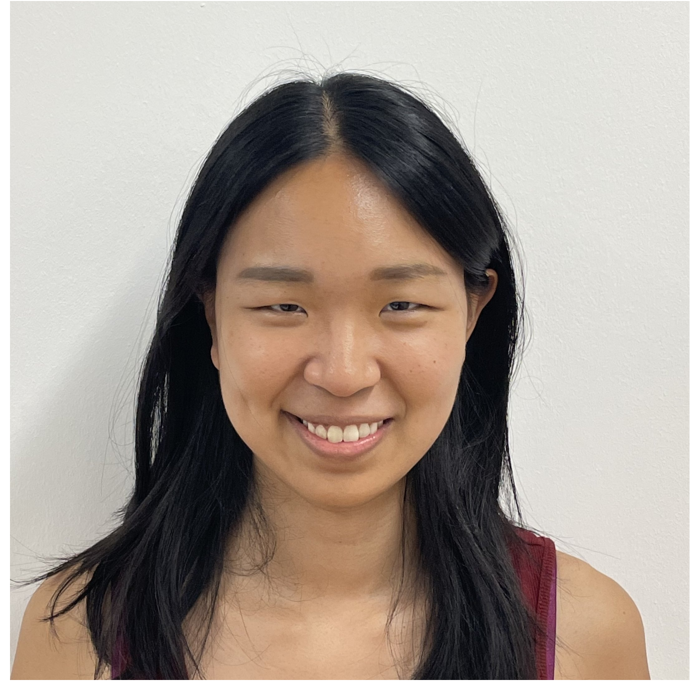
Freie Universität Berlin

Joint work with…



Pseudomagic quantum states

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Notions of nonstabilizerness, or "magic", quantify how non-classical quantum states are in a precise sense: states exhibiting low nonstabilizerness preclude quantum advantage. We introduce 'pseudomagic': ensembles of quantum states that, despite low nonstabilizerness, are computationally indistinguishable from those with high nonstabilizerness. Previously, such computational indistinguishability has been studied with respect to entanglement, introducing the concept of pseudoentanglement. However, we demonstrate that pseudomagic neither follows from pseudoentanglement nor implies it. In terms of applications, the study of pseudomagic offers fresh insights into the theory of quantum chaos: it uncovers states that, even though they originate from non-chaotic unitaries, remain indistinguishable from random chaotic states to any physical observer. Additional applications include new lower bounds on state synthesis problems, property testing protocols, and implications for quantum cryptography. Our findings suggest that nonstabilizerness is a 'hide-able' characteristic of quantum states: some states are much more magical than is apparent to the (computationally-bounded) observer. From the physics perspective, our study supports the idea that only quantities which can be measured in a computationally efficient manner are physically significant.

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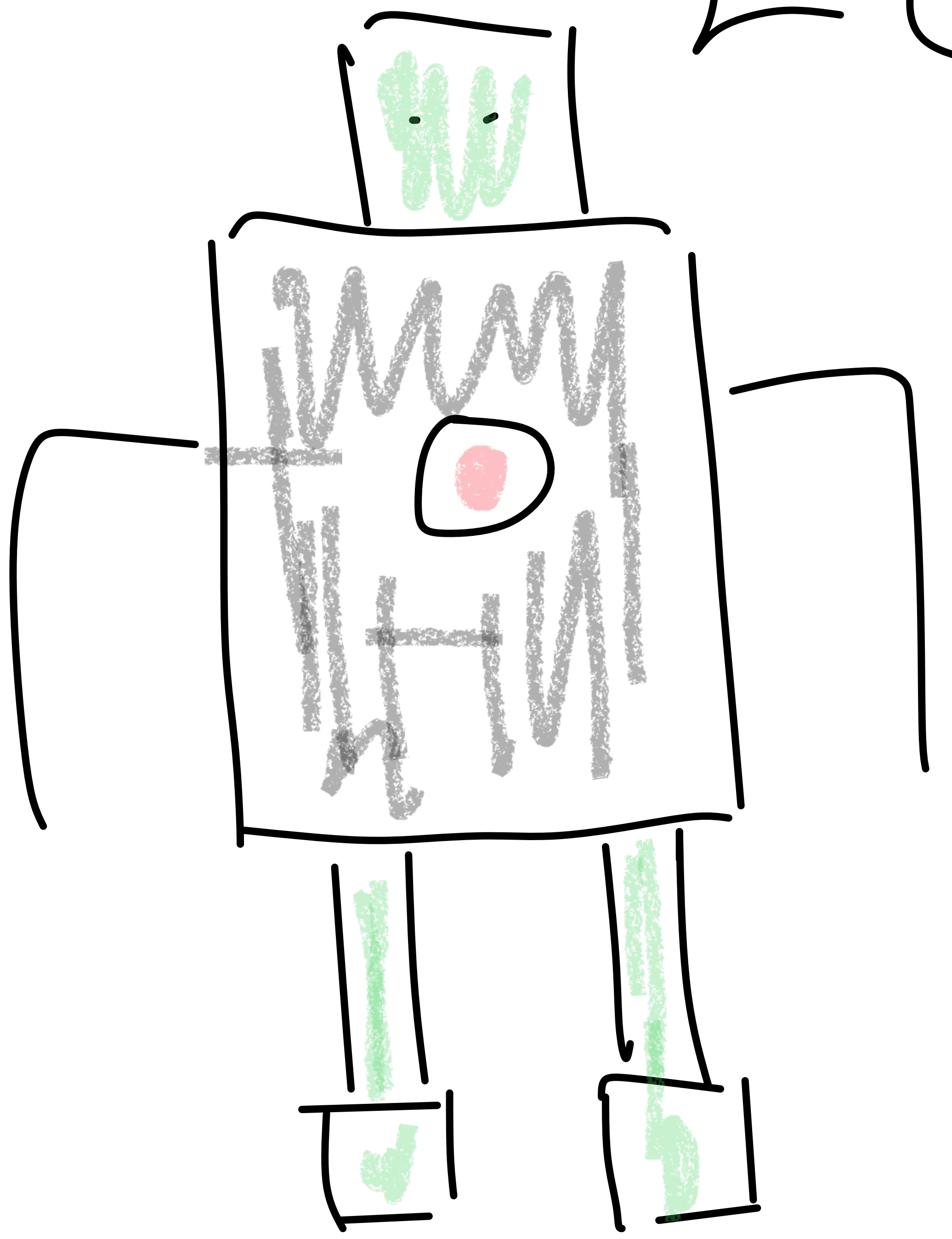
OUTLINE

- MOTIVATION.
- PSEUDORANDOM STATES.
- MAGIC IN QUANTUM STATES.
- CHAOS IN QUANTUM EVOLUTIONS.
- PSEUDOMAGIC.

TAKE-HOME:

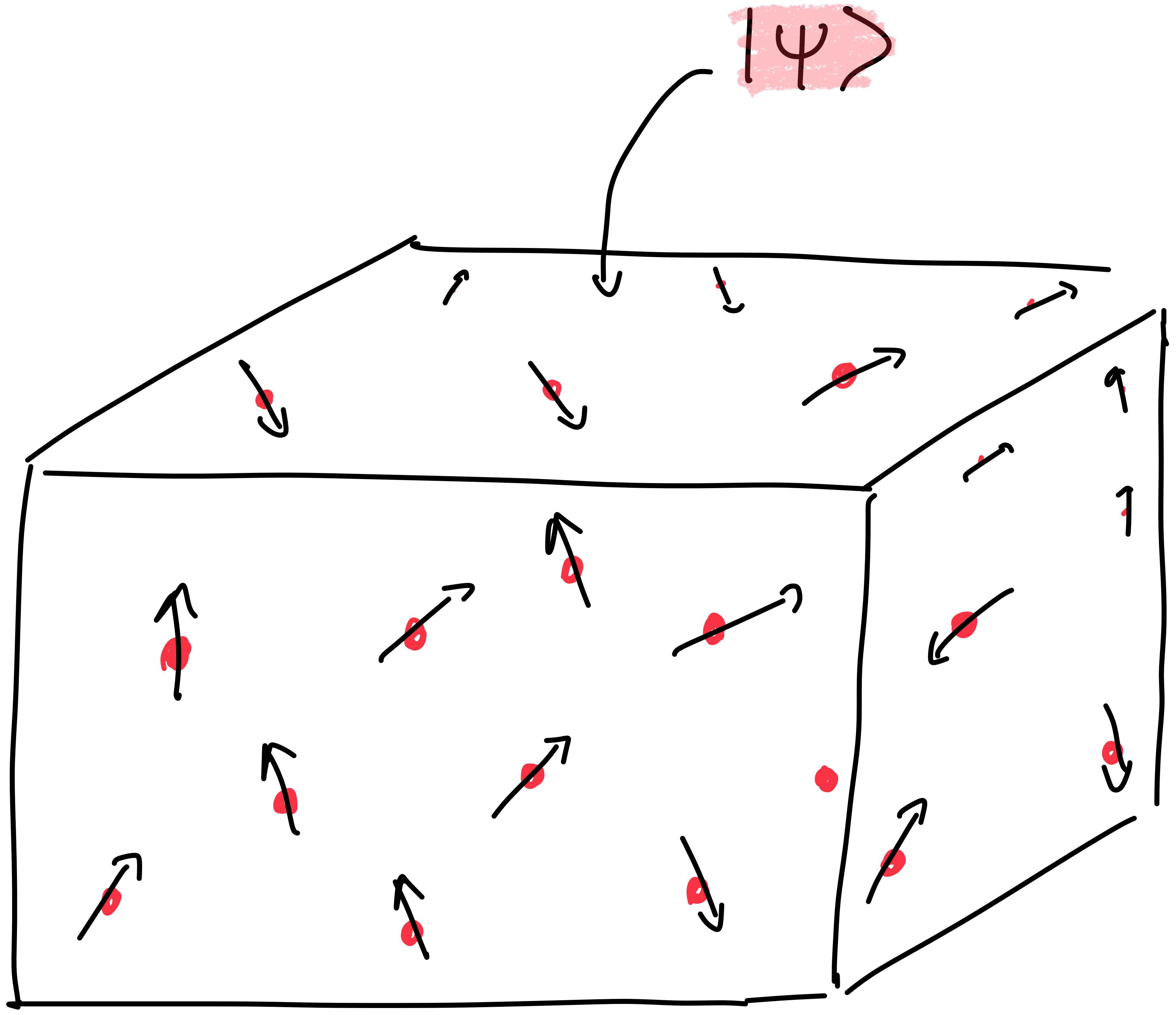
NO PHYSICAL OBSERVER CAN PROVABLY DISTINGUISH CHAOTIC STATES FROM NON-CHAOTIC ONES.



MOTIVATION

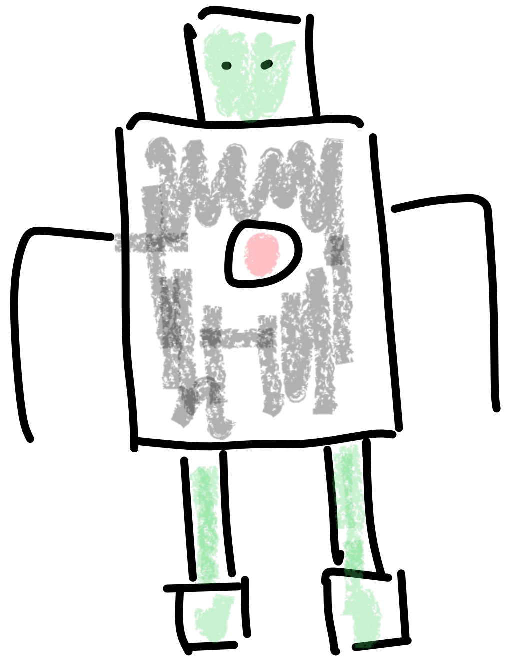
- LET US CONSIDER A QUANTUM **MANY-BODY** SYSTEM

$$\begin{cases} n: \text{ \# PARTICLES WITH SPIN } 1/2 \longmapsto \text{"QUBITS"} \\ |\psi\rangle: \text{ QUANTUM STATE OF THE MB SYSTEM.} \end{cases}$$



QUESTION:

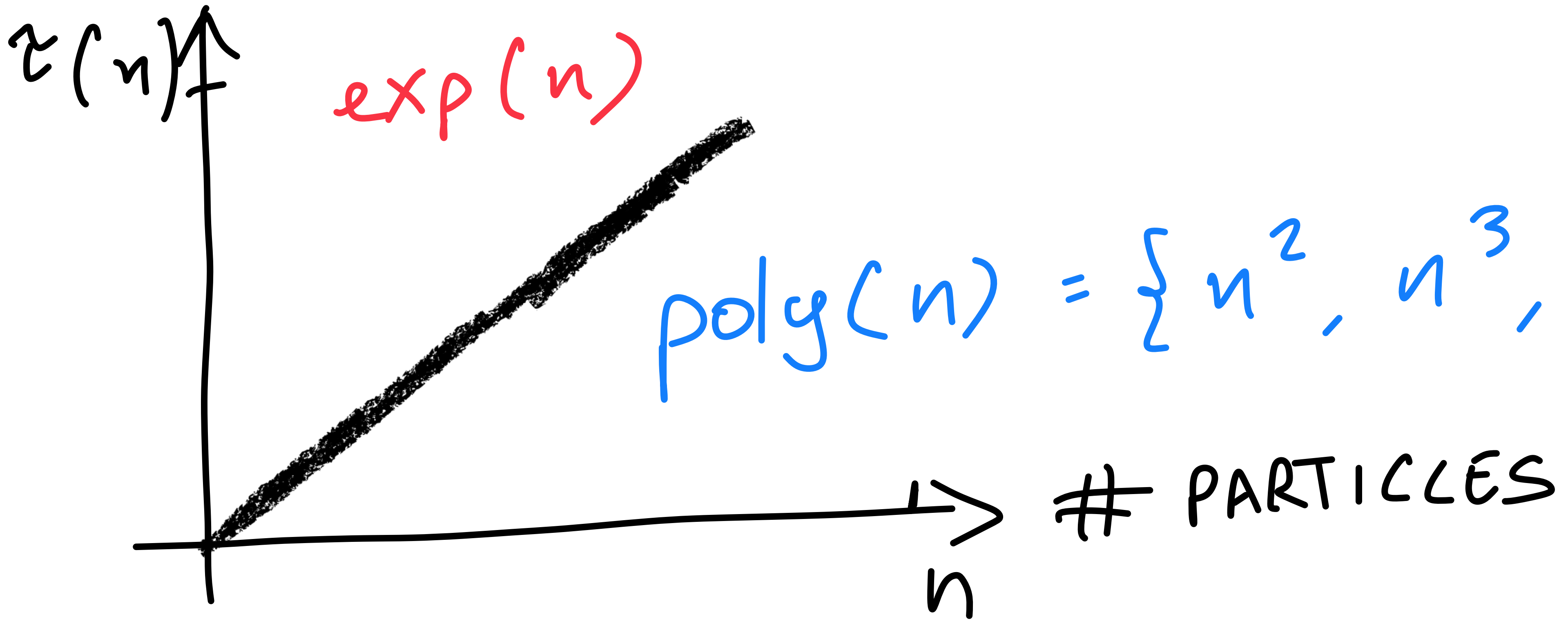
GIVEN AN OBSERVABLE $\hat{O}$.
WHAT IF MEASURING $\langle \hat{O} \rangle_\psi$
TAKES MORE TIME THAN
AGE OF THE UNIVERSE ?!



BORROWING NOTIONS FROM CS

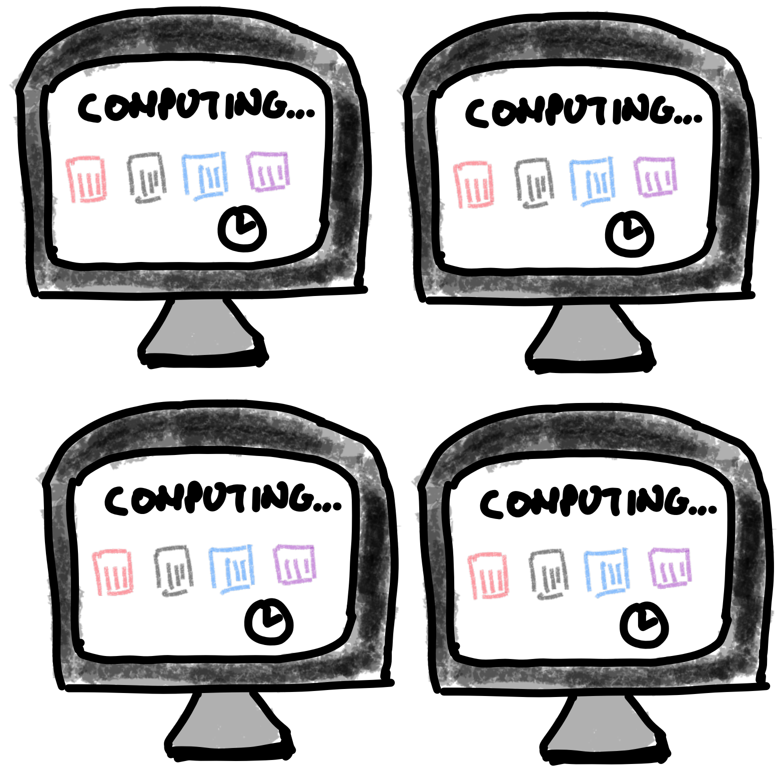
- EVERY MEASUREMENT SCHEME IS A (QUANTUM) ALGORITHM
- COMPUTATION TIME ⟼ TIME COMPLEXITY $\tau(n)$

  FUNCTION OF THE # PARTICLES n
- (FUZZY) BLACK LINE IS THE BOUNDARY: POSSIBLE / IMPOSSIBLE



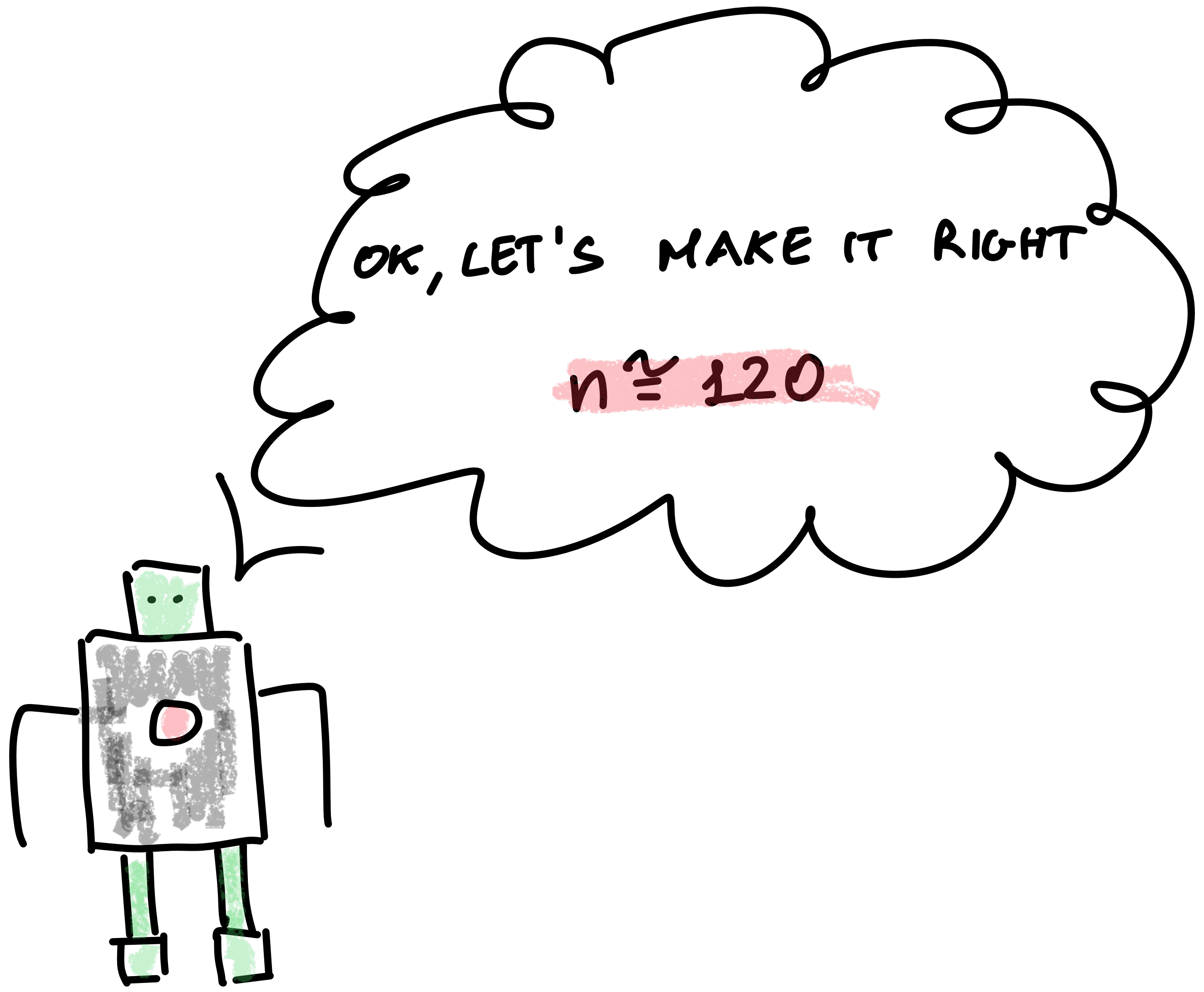
QUESTION: WHAT IS n SUCH THAT $\exp(n)$ = AGE OF THE UNIVERSE ?!

AGE OF THE UNIVERSE ⟼ 10^{18} SECONDS



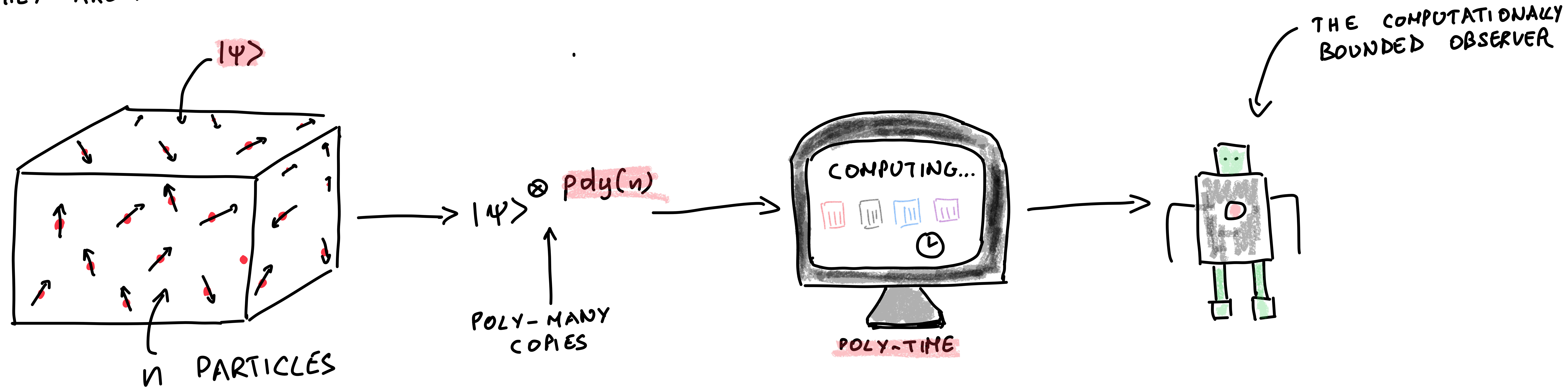
FASTEST SUPER-COMPUTER

$10^{18}\ \frac{\text{OPERATIONS}}{\text{SECONDS}}$



THE COMPUTATIONALLY BOUNDED OBSERVER

- HE/SHE/THEY ARE ALLOWED TO USE A POLY-TIME BOUNDED MEASUREMENT SCHEME



CONSEQUENCE:

- THERE EXIST STATES WHICH ARE PSEUDORANDOM $\longrightarrow$ CANNOT BE DISTINGUISHED BY PURELY RANDOM STATES

EXAMPLE : SUBSET STATES

$|x\rangle$: BITSTRING eg $x = 0101\ldots1$

2^n POSSIBLE BITSTRINGS $x \in \{0,1\}^n$

SUBSET $S \subseteq \{0,1\}^n$

$$\boxed{|\psi_S\rangle = \frac{1}{\sqrt{|S|}} \sum_{x \in S} |x\rangle}$$

$\Longrightarrow$ IF SIZE OF S IS $|S| = \mathrm{poly}(n)$

$\Downarrow$

$|\psi_S\rangle$ PSEUDORANDOM

MAGIC IN QUANTUM STATES

- MAGIC IS THE "FUEL" THAT PREVENTS CLASSICAL SIMULABILITY OF QUANTUM STATES.

- IT CAN BE DEFINED AS ENTROPY THROUGH PAULI MATRICES

$$I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix};\quad X = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix};\quad Y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix};\quad Z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \longrightarrow \text{MULTI-PARTICLE} \left\{ \underset{1}{X} \otimes \underset{2}{Y} \otimes \underset{3}{Z} \otimes \underset{\cdots}{\cdots} \otimes \underset{n}{X} \right. \equiv P \Big\}$$

SINGLE PARTICLE...

PAULI GROUP
(4^n ELEMENTS)

- STATES CAN BE EXPRESSED THROUGH EXP. VALUE OF PAULI

DENSITY MATRIX

PAULI

$$\boxed{|\psi\rangle\langle\psi| = \sum_P c_P \, P}$$

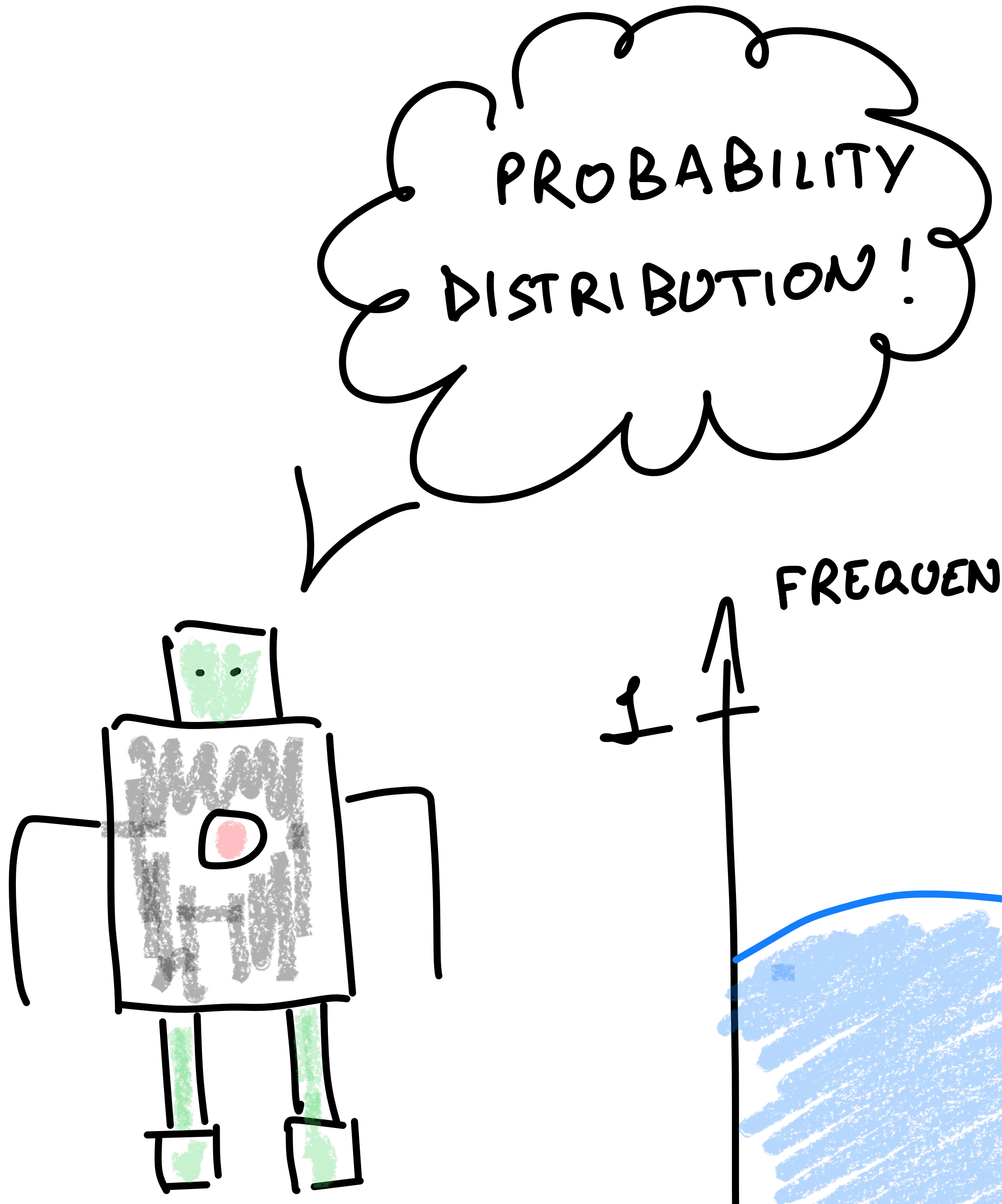
EXP. VALUES
$$c_P \equiv \frac{\langle\psi|P|\psi\rangle}{2^n}$$

- EXP. VALUES SATISFY...

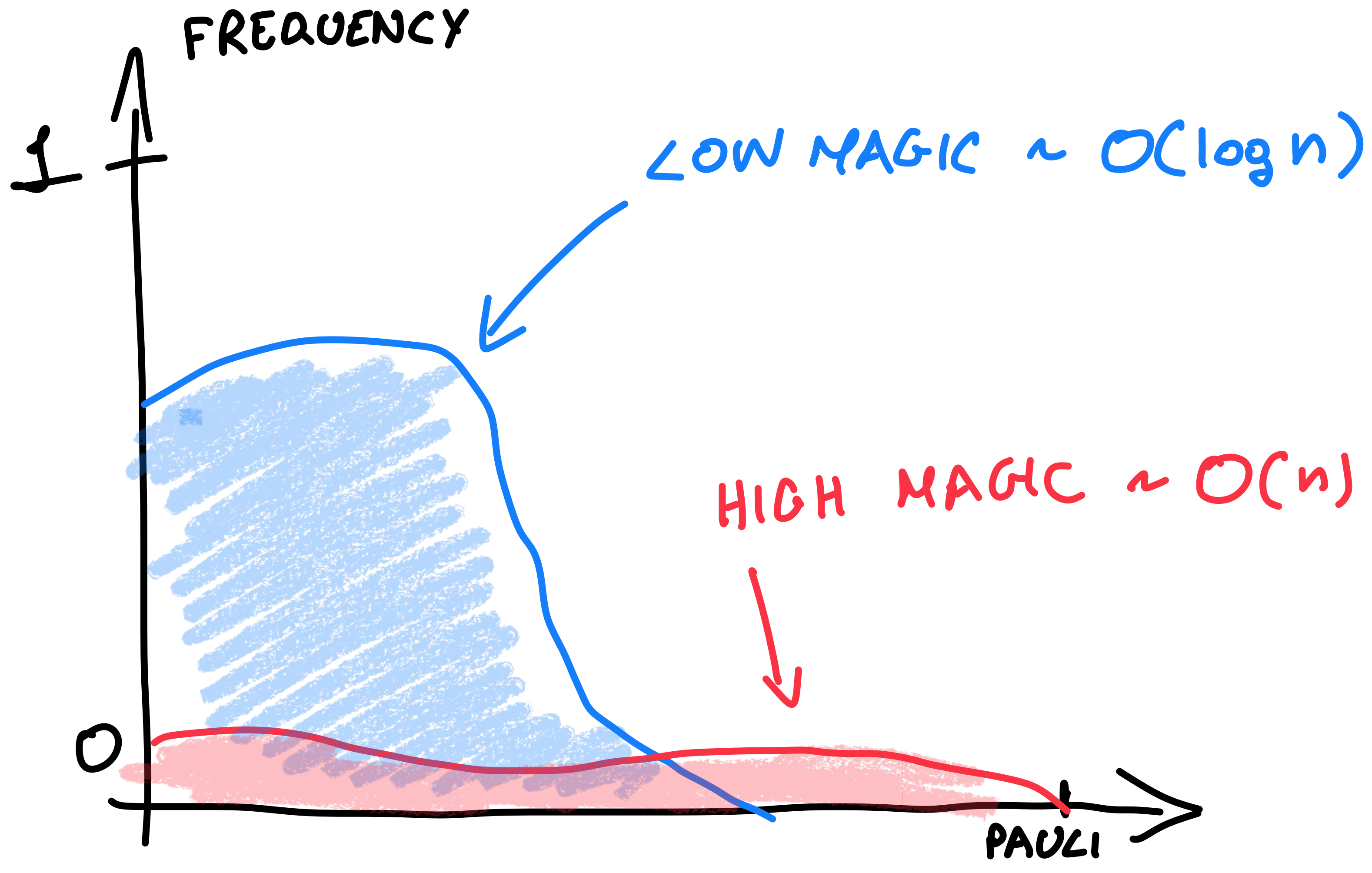
1) $2^n \sum_P c_P^2 = 1$ NORMALIZED

2) $c_P^2 \geq 0$ POSITIVE

$\Longrightarrow$



MAGIC $M(|\psi\rangle)$



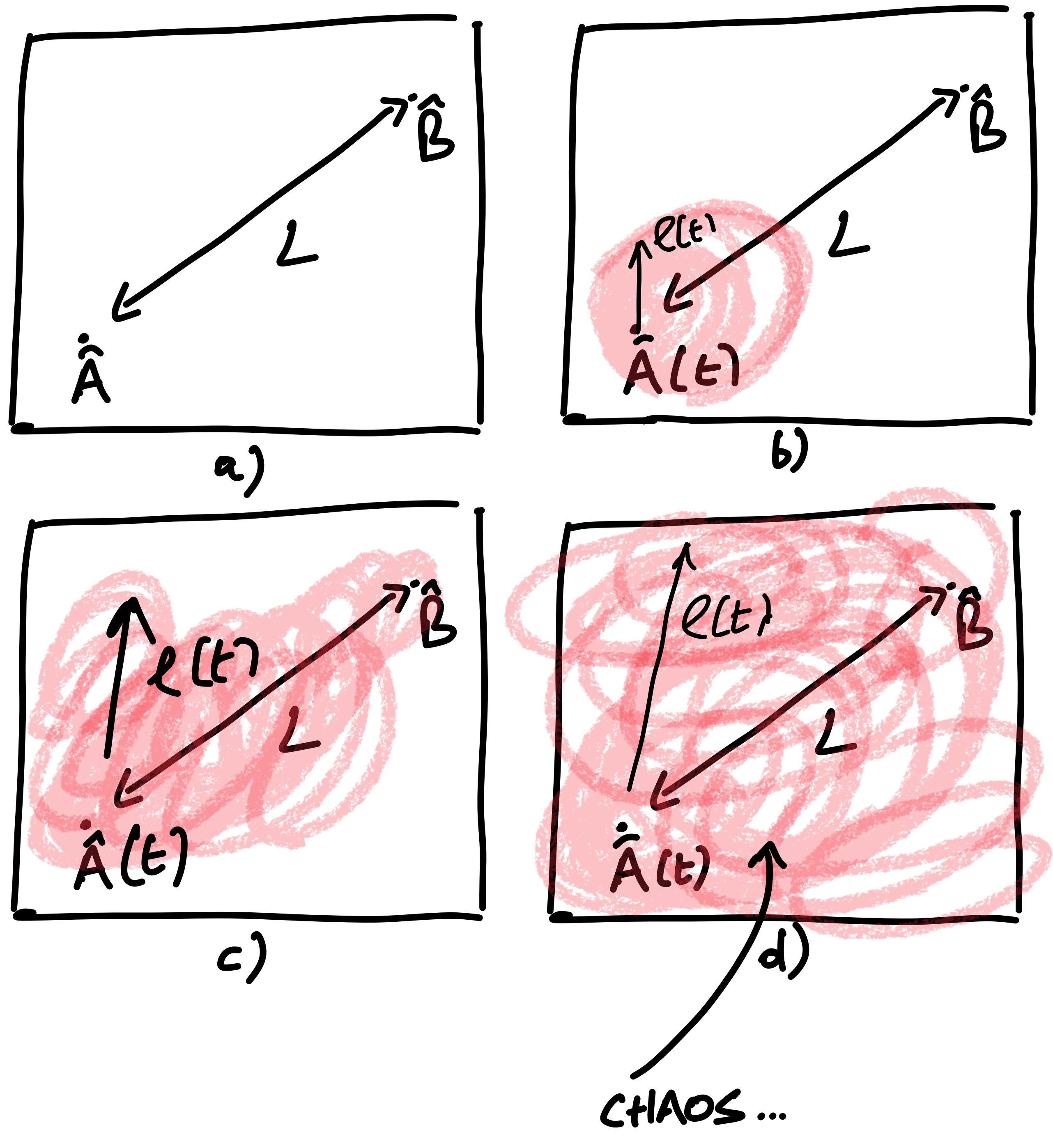
QUANTUM CHAOS

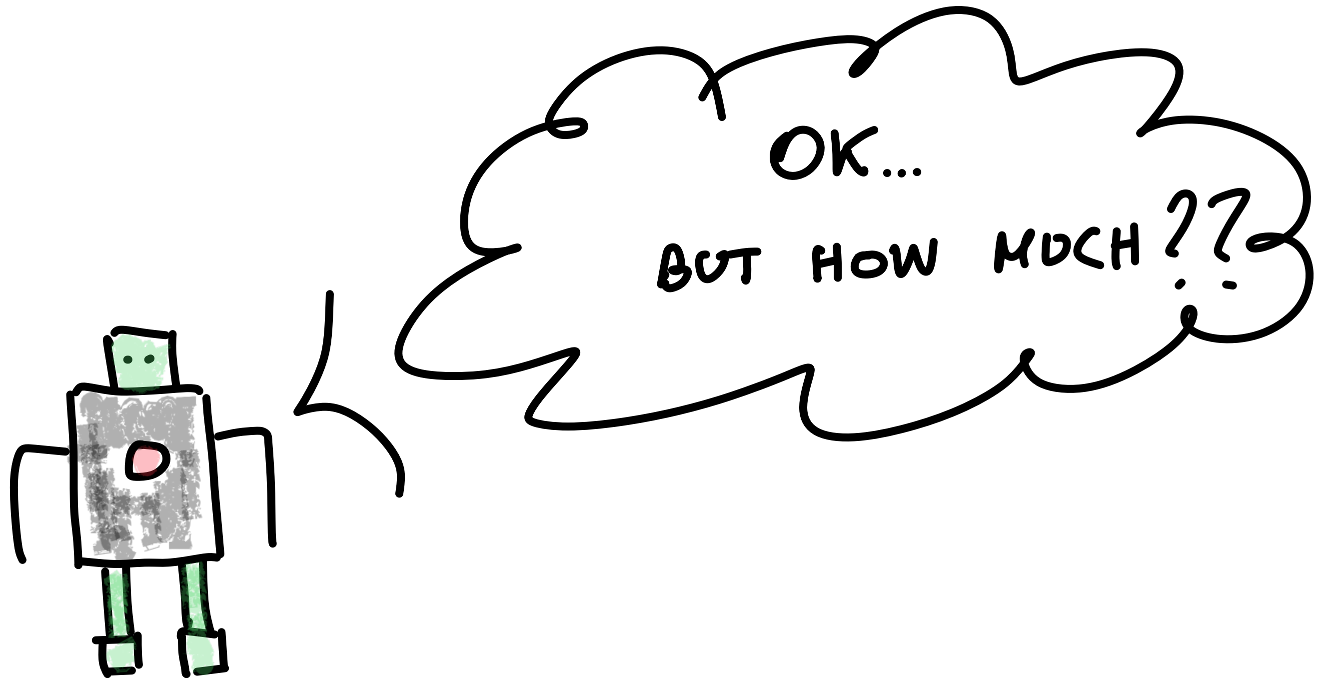
- MODERN APPROACH TO QUANTUM CHAOS $\longrightarrow$ QUANTUM BUTTERFLY EFFECT

a) TWO LOCALIZED OBSERVABLES A, B. $\Longrightarrow$ $[A,B]$ COMMUTE!

b) $\hat{A}$ EVOLVED IN TIME WITH $e^{-i\hat{H}t}$ $\longrightarrow$ $A(t) = e^{i\hat{H}t} A e^{-i\hat{H}t}$

c) $\hat{A}(t)$ REACHES $\hat{B}$ $\Longrightarrow$ $[\hat{A}(t), B] \neq 0$ DO NOT COMMUTE!





$\Longrightarrow$

$\|[A(t), B]\|_2$ $\longrightarrow$ NORM OF THE COMMUTATOR

$$\|[\hat{A}(t), \hat{B}]\|_2 = \sqrt{1 - \mathrm{OTOC}(t)}$$

- $\mathrm{OTOC}(t) \equiv$ OUT-OF-TIME-ORDER CORRELATOR

$$\boxed{\mathrm{OTOC}(t) \equiv \frac{1}{2^n} \mathrm{Tr}\left[A(t)\, B\, A(t)\, B\right]}$$

MODERN DEF. CHAOTIC EVOLUTION

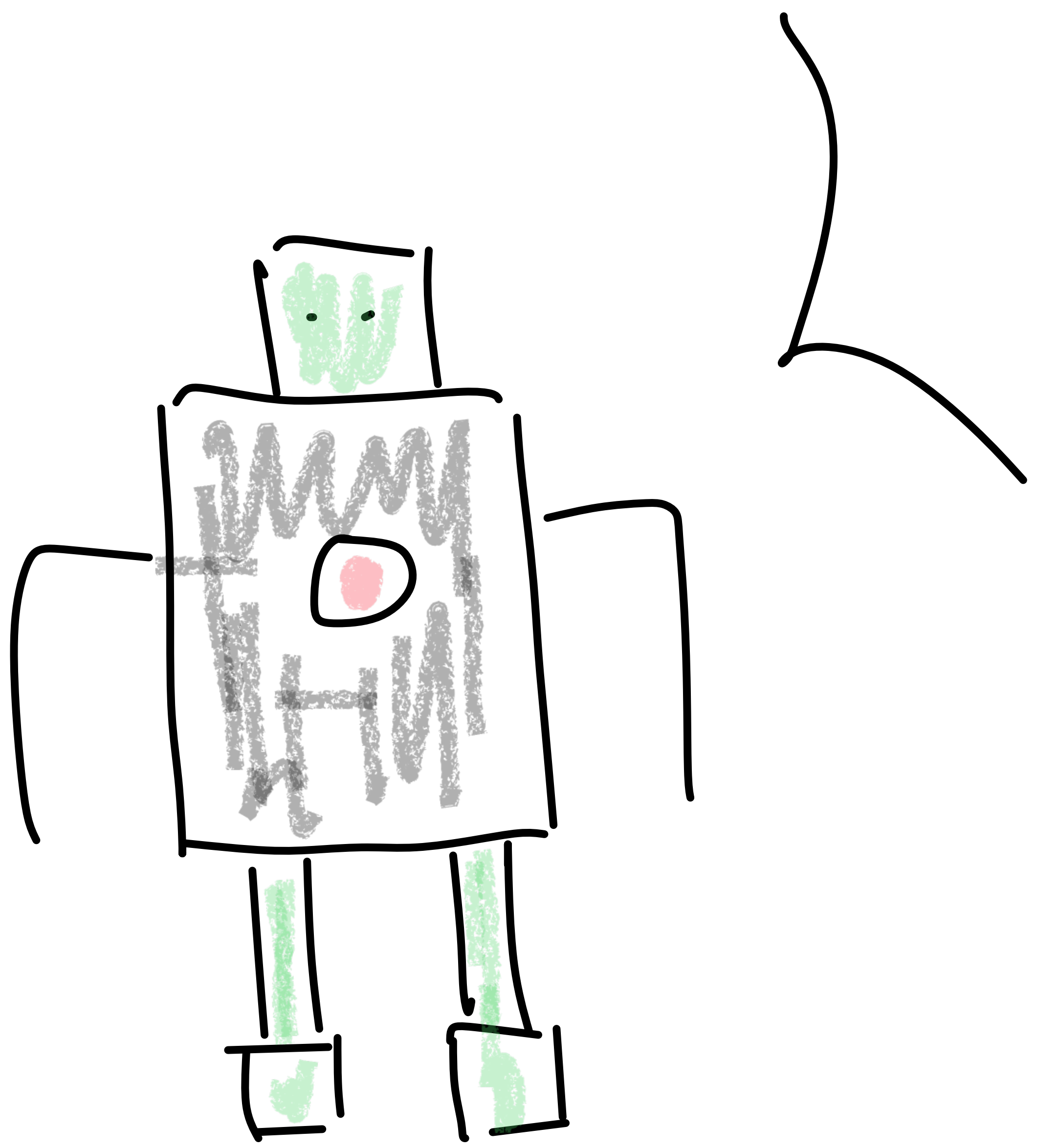
$\boxed{e^{-i\hat{H}t}}$

CHAOS: $\mathrm{OTOC}(t) = \dfrac{1}{\exp(n)}$

NON-CHAOS: $\mathrm{OTOC}(t) = \dfrac{1}{\mathrm{poly}(n)}$

MAGIC IN QUANTUM STATES = CHAOS IN QUANTUM DYNAMICS

- $\hat{H}$: HAMILTONIAN.
- $|\psi_0\rangle$: INITIAL QUANTUM STATE.
- $|\psi(t)\rangle = e^{-i\hat{H}t}|\psi_0\rangle$ EVOLUTION



MAGIC AND CHAOS ARE RELATED!!

$$\boxed{M(|\psi(t)\rangle) = -\log \mathrm{OTOC}(t)}$$

CHAOTIC DYNAMICS PRODUCES ↗ HIGH MAGIC

$$\mathrm{OTOC}(t) = \frac{1}{\exp(n)} \Longrightarrow M(|\psi_t\rangle) = O(n)$$

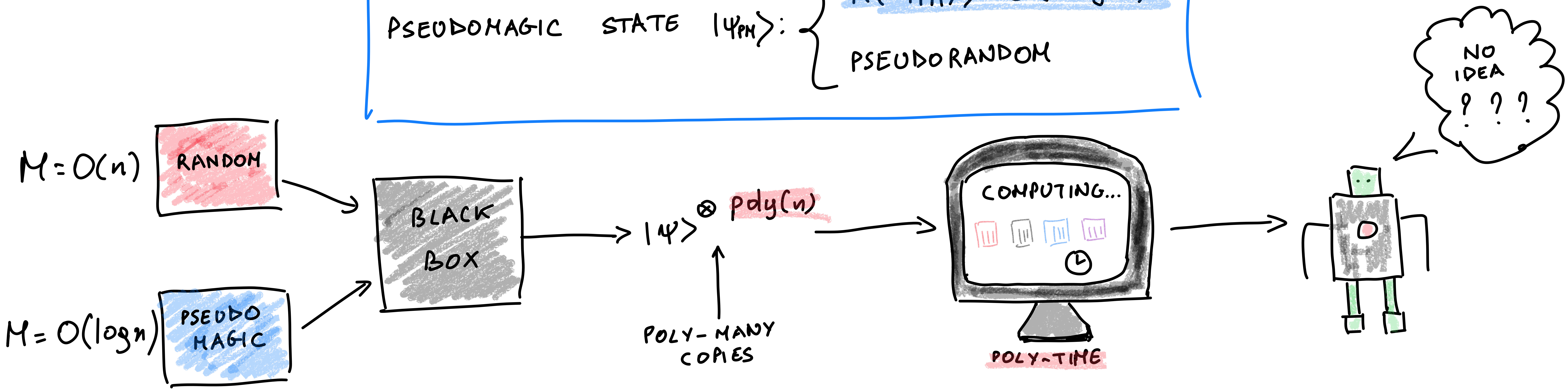
NON-CHAOTIC DYNAMICS LOW MAGIC...

$$\mathrm{OTOC}(t) = \frac{1}{\mathrm{poly}(n)} \Longrightarrow M(|\psi_t\rangle) = O(\log(n))$$

PSEUDOMAGIC QUANTUM STATES

- MAGIC RANDOM STATES $M(|\psi_{RANDOM}\rangle) = O(n)$

PSEUDOMAGIC STATE $|\psi_{PM}\rangle$: $\begin{cases} M(|\psi_{PM}\rangle) = O(\log n) \\ \text{PSEUDORANDOM} \end{cases}$



CONSEQUENCES:

1) TOO MUCH "FUEL" IS MAYBE USELESS $O(\log n) \approx O(n)$

2) CHALLENGE QUANTUM CHAOS

! "CHAOTIC" STATES $\approx$ NON-CHAOTIC STATES !

FOR ANY COMPUTATIONALLY BOUNDED OBSERVER

Thanks.