Inferring the composition of Ultra-High Energy Cosmic Rays

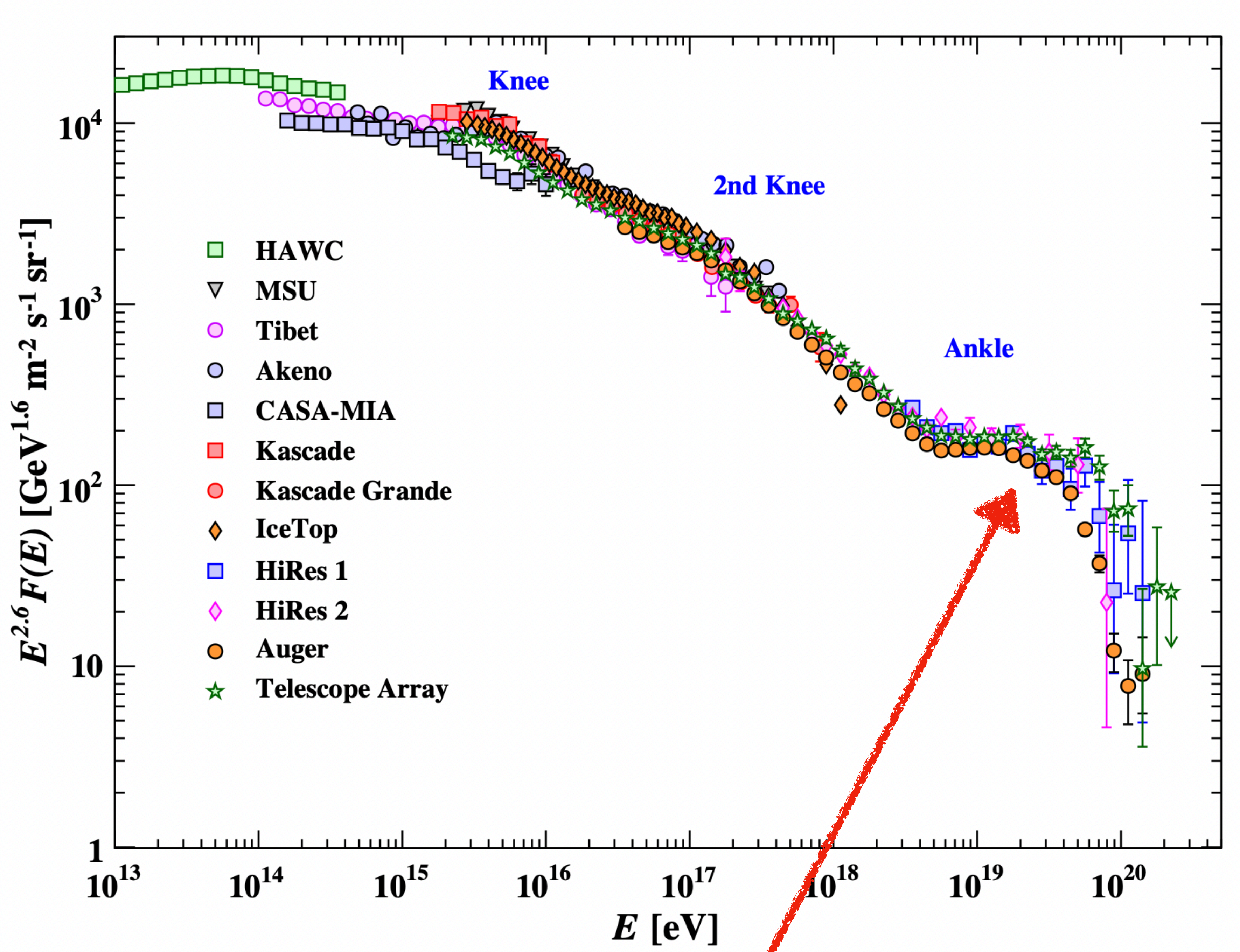
Michele Tammaro

@ Re[Incontri] di Fisica Partenopea, 20/12/2023

with B. Bortolato and J.F. Kamenik (2212.04760, 2304.11197 and more to come…)

Istituto Nazionale di Fisica Nucleare
SEZIONE DI FIRENZE

Ultra High-Energy Cosmic Rays (UHECR)



$$\phi \sim 10^{-2}\ \mathrm{km}^{-2}\ \mathrm{yr}^{-1}$$

Where?

Astrophysical sources of UHECR

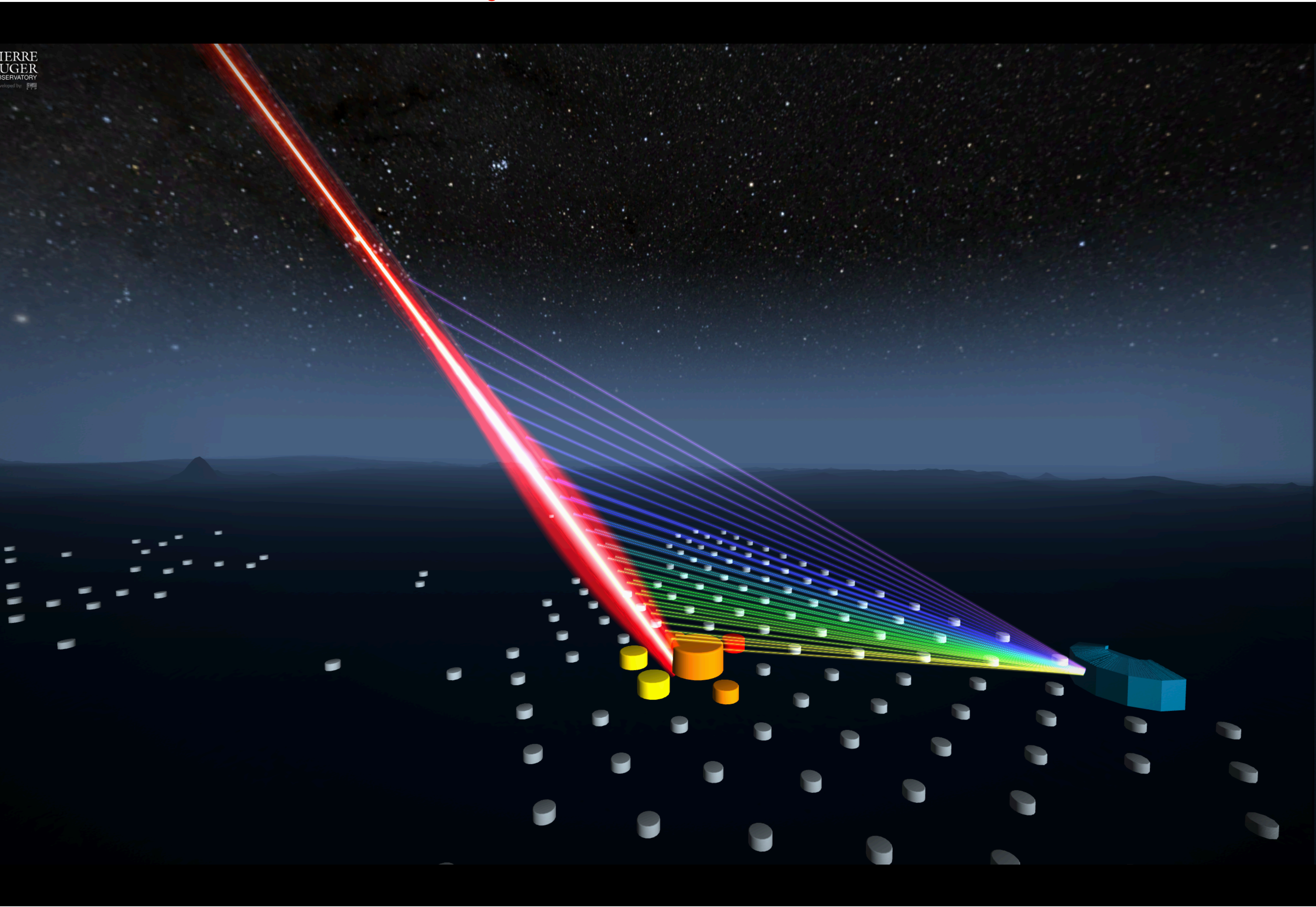
How?

Acceleration mechanisms

What?

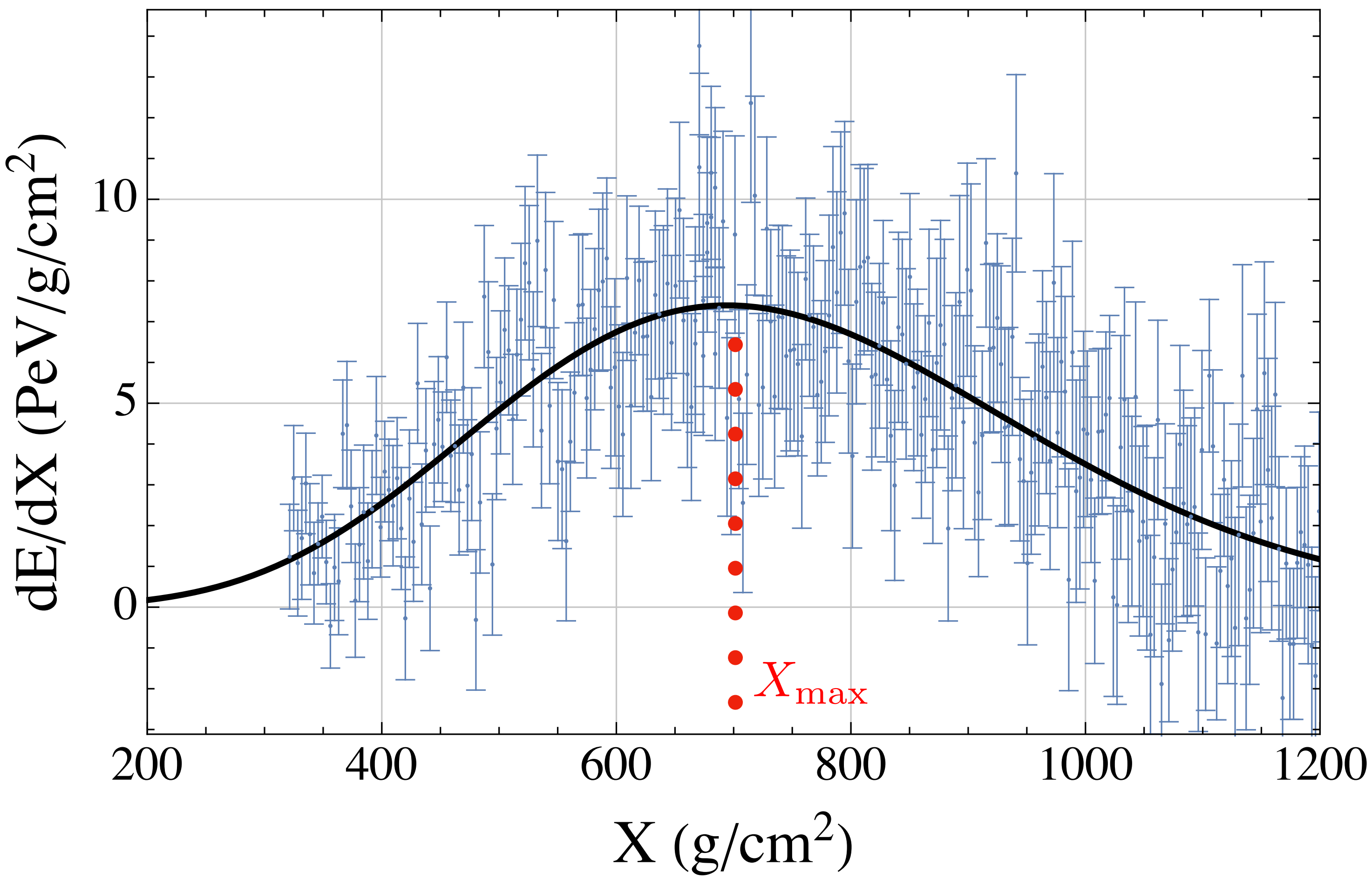
Mass composition

Hybrid Showers

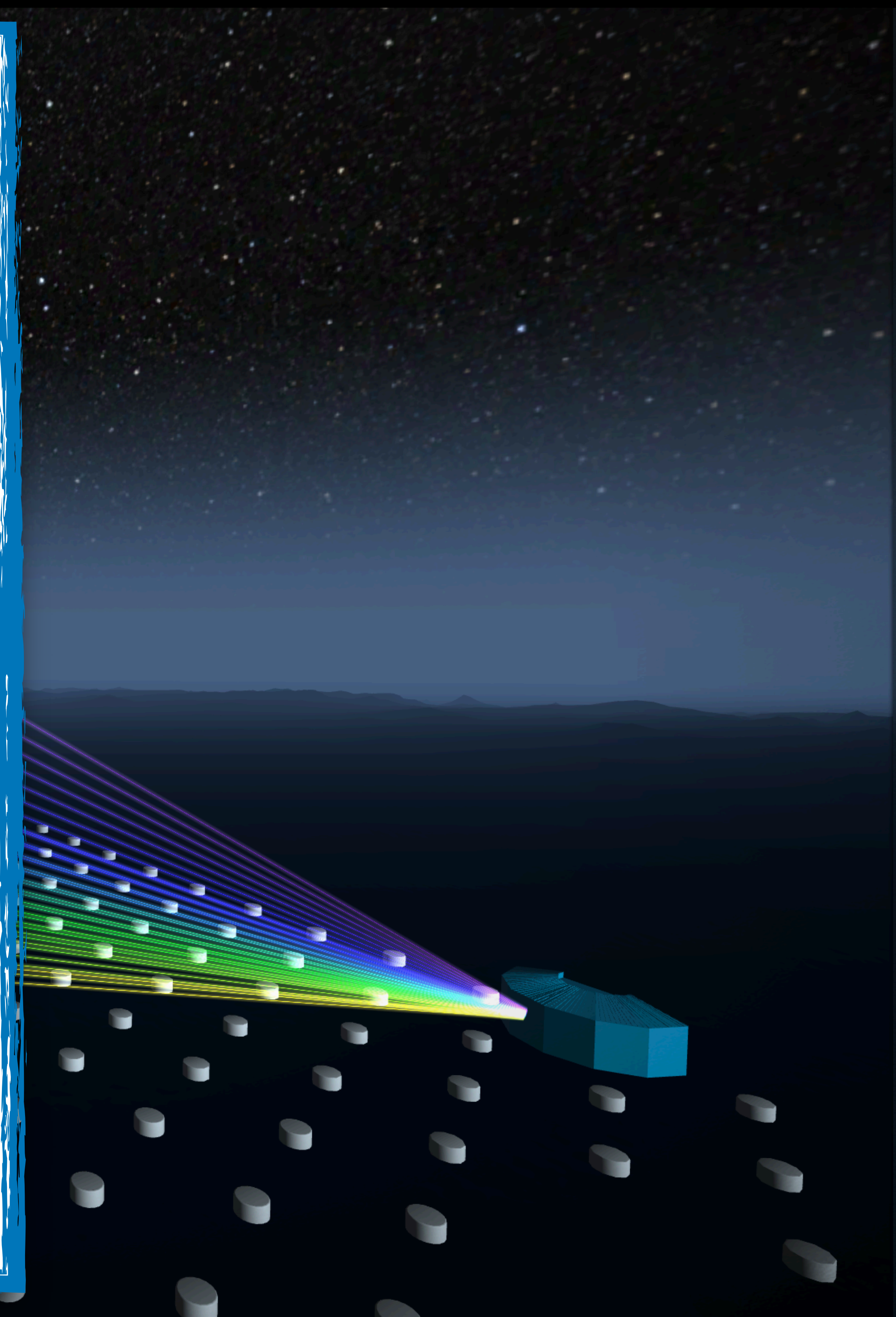


Hybrid Showers

$$X_{\text{ground}} \sim 1200\ \text{g/cm}^2$$

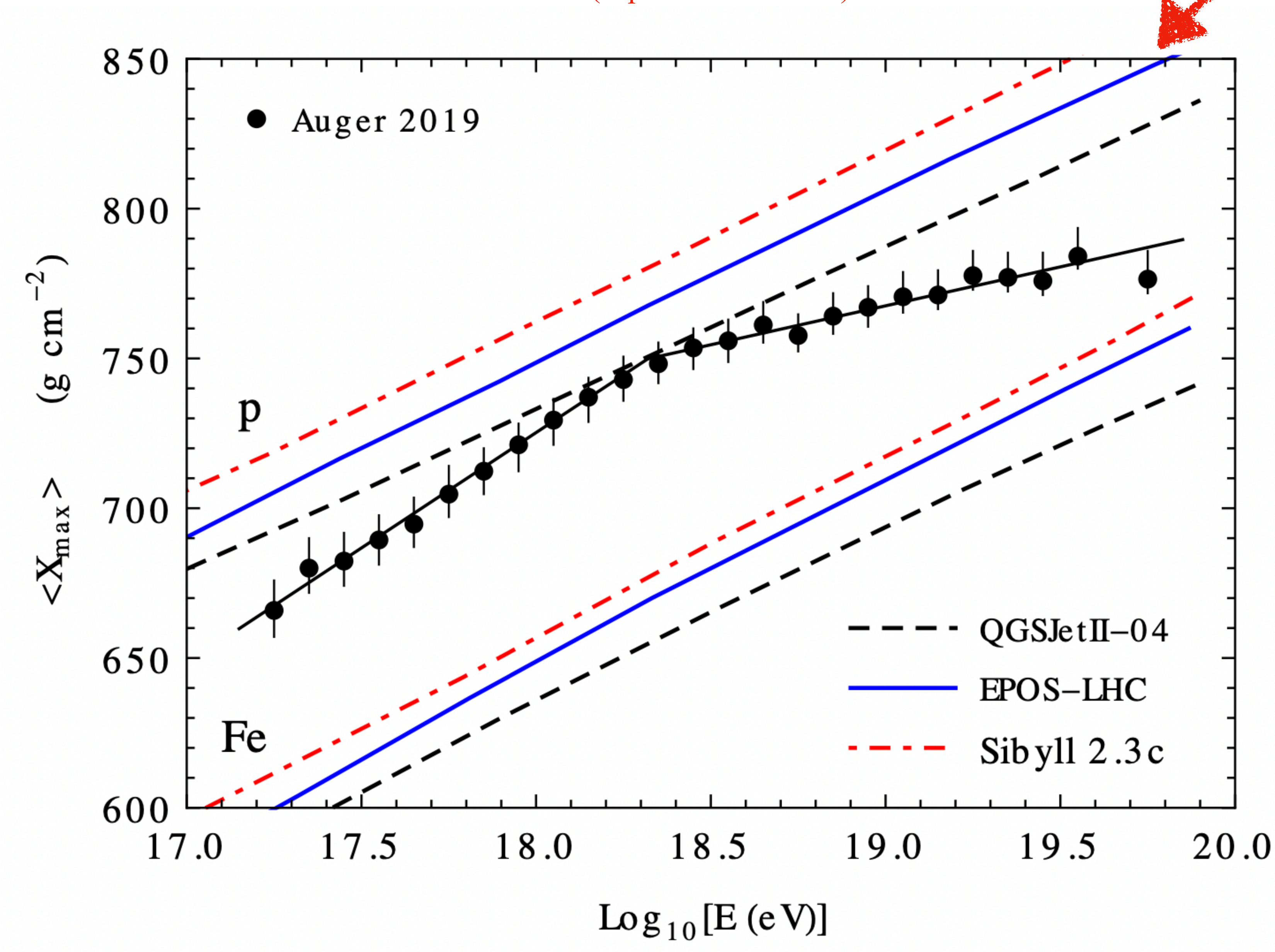


$$\langle X_{\text{max}} \rangle \propto \log E - \log A$$



Different models, which one to use?
(Not in this talk, but interesting problem)

(Lipari: 2012.06861)



Simulations perfomed with CORSIKA (https://www.iap.kit.edu/corsika/)

Auger data seems to indicate heavier primaries at higher energies

What has been done

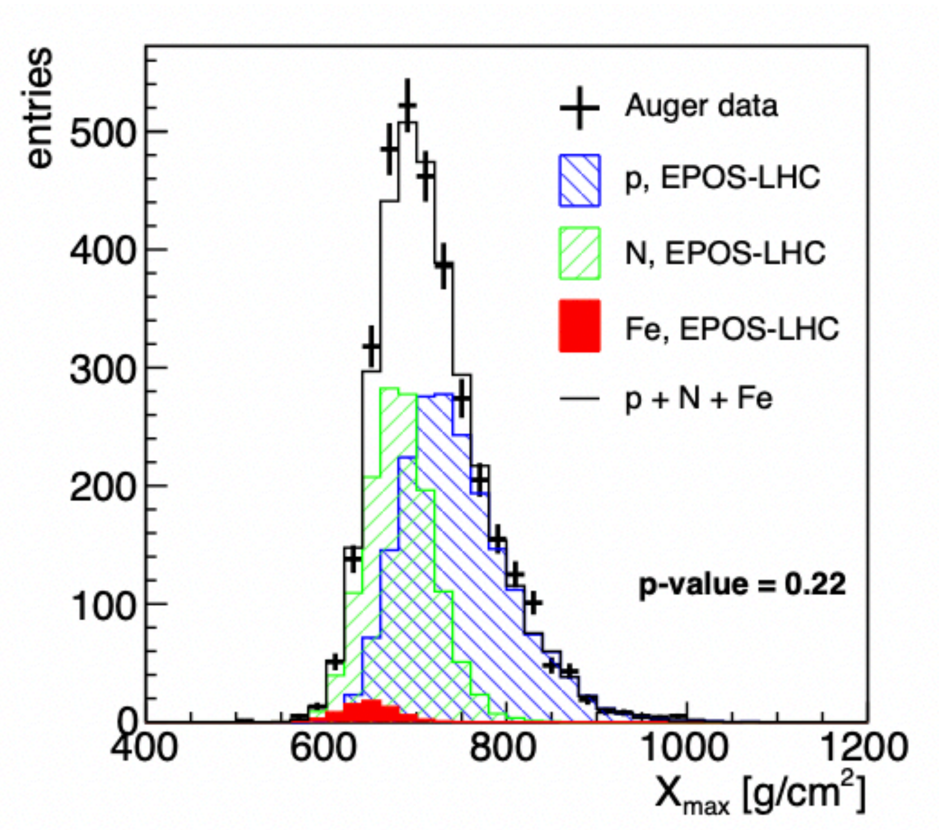
Fit $X_{\max}$ distribution with mixture

$$(p,\ He,\ N,\ Fe)$$

Bin and count

(Arsene, Sima: 2001.02667)
(Auger Coll.: 1409.5083)
(Lipari: 2012.06861)

$$p \sim 65\%, \quad N \sim 35\%$$



What we propose

$$w = (w_p,\ w_{He}, \ldots,\ w_{Fe}) \qquad \sum_i^{26} w_i = 1 \qquad \text{25 free parameters}$$

Decompose distribution in moments ⟶ Unbinned likelihood

Faster computation with Nested Sampling

What has been done

Fit $X_{\max}$ distribution with mixture

$(p,\ He,\ N,\ Fe)$

Bin and count

(Arsene, Sima: 2001.02667)
(Auger Coll.: 1409.5083)
(Lipari: 2012.06861)

$p \sim 65\%, \quad N \sim 35\%$

What we propose

$$w = (w_p,\ w_{He}, \ldots,\ w_{Fe}) \qquad \sum_i^{26} w_i = 1$$

25 free parameters

Decompose distribution in moments ⟶ Unbinned likelihood

Faster computation with Nested Sampling

Problem is reduced to fit vector of moments

Data

Simulations

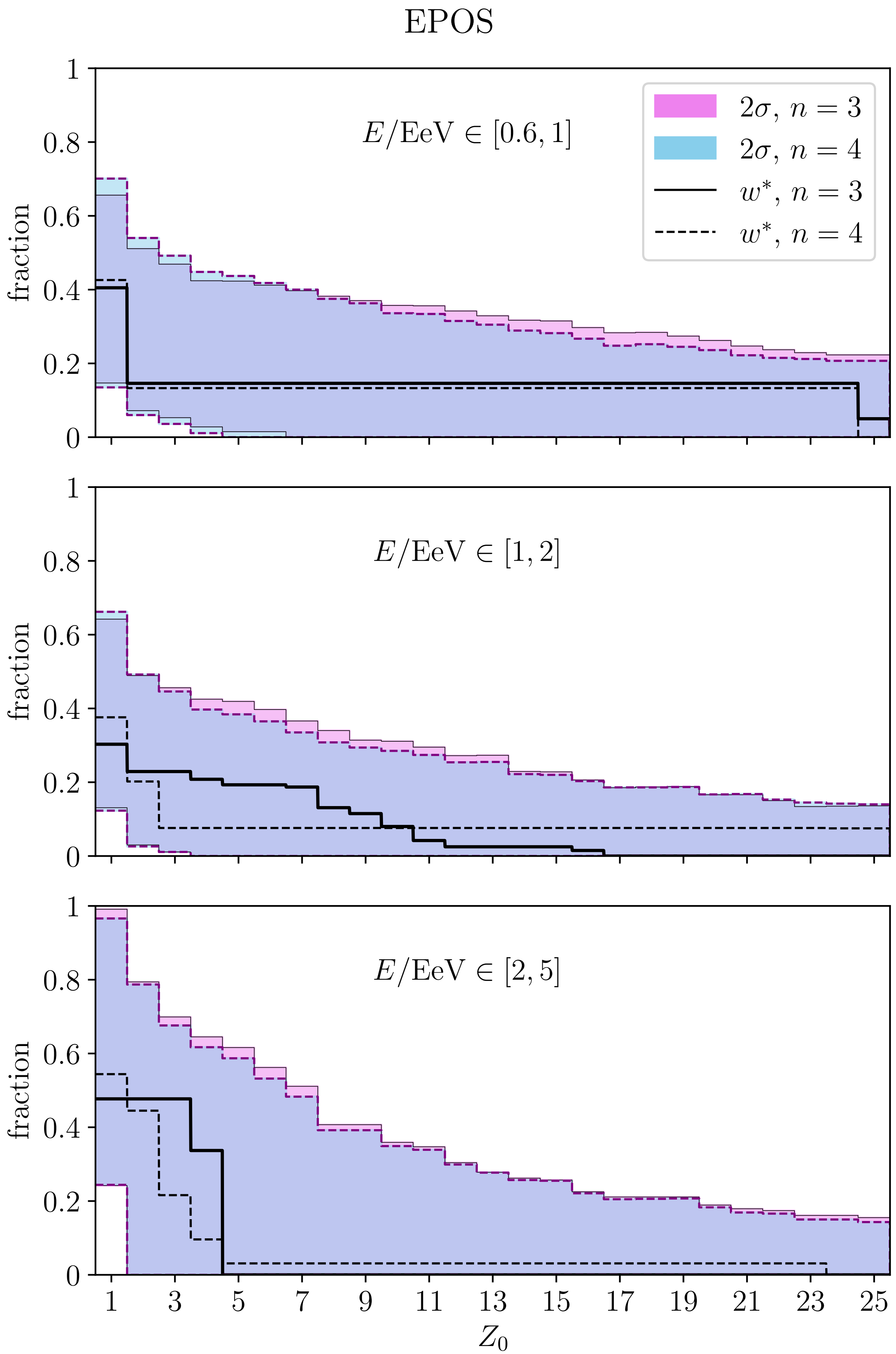
$$\tilde{\mathcal{L}}(z, w) = \mathcal{N}_n\left(z \mid \tilde{\mu}, \tilde{\Sigma}\right) \times \mathcal{N}_n\left(z \mid \mu_w, \Sigma_w\right)$$

$$\mathcal{L}(w) = \int \tilde{\mathcal{L}}(z, w)\ \mathrm{d}^n z$$

Assume a flat prior (Dirichlet distribution) and Maximize log-likelihood

$$\mathcal{P}(w) = \frac{\mathcal{L}(w)\ \mathrm{Dir}(w, \alpha)}{\int \mathcal{L}(w)\ \mathrm{Dir}(w, \alpha)\ \mathrm{d}^D w}$$

Full (cumulative) composition - EPOS



Fraction of elements heavier than Z_0

Can exclude 100% proton compositions

Results are unchanged increasing the number of features

N.B.: publicly available data is very limited (few hundreds of events)

Including ground data

Hybrid

$$(x_i, y_i)_{i=1,\dots,N}$$

FD → x_i ; SD → y_i

Correlation $\rho(x,y) = \frac{\mathrm{Cov}(x,y)}{\sigma(x)\sigma(y)}$

Non-Hybrid

$$(\hat{x}(\hat{y}_j), \hat{y}_j)_{j=1,\dots,M}$$

How much information is added?

Assumption:
the underlying distribution of events
is the same for both sets

Including ground data

$$\mathcal{X}_\ell = \{(x_i, y_i)\}_{\ell=1,\dots,B}$$

$B =$ number of bootstrapping samples

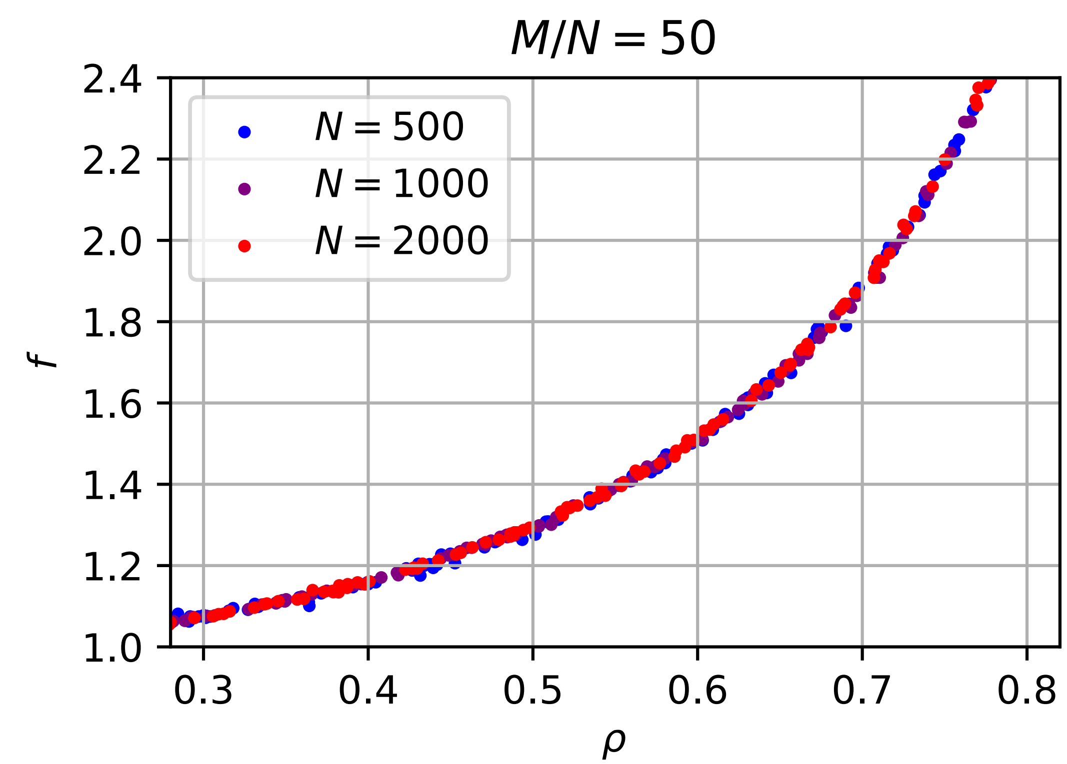
Get joint distributions $P_\ell(x, y)$

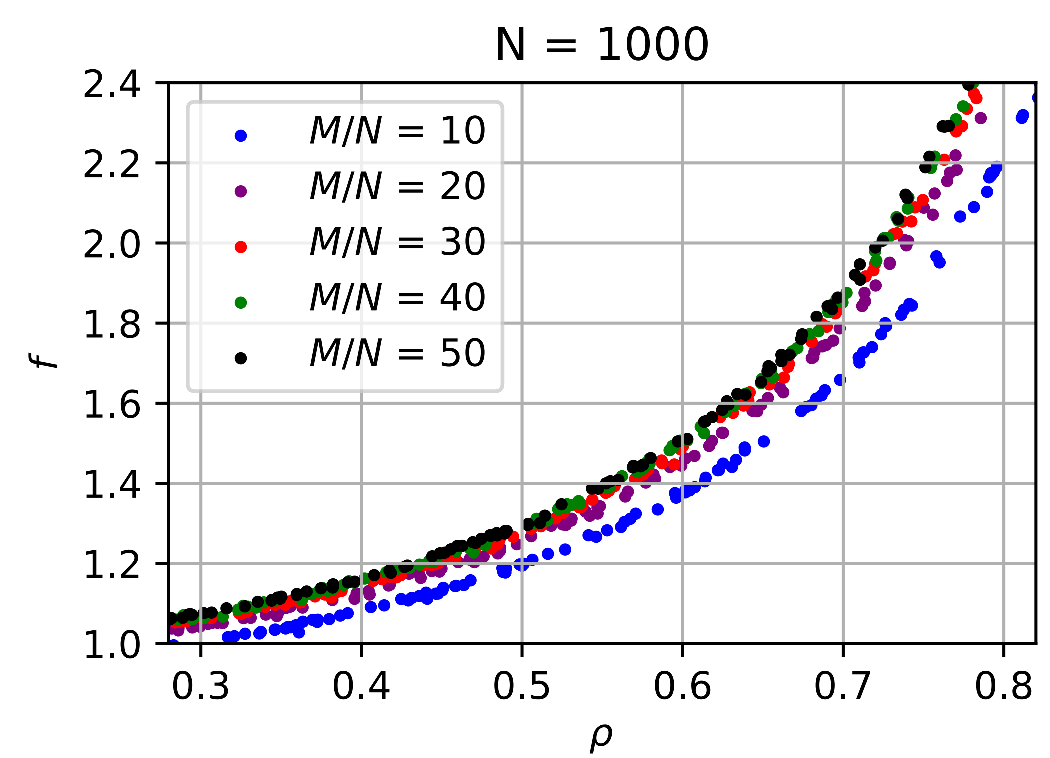
$$\mathcal{X}_\ell^{\text{inf}} = \{(\hat{x}(\hat{y}_j), \hat{y}_j)\}_{\ell=1,\dots,B}$$

$$\mathcal{X}_l^{\text{comb}} = \mathcal{X}_\ell \cup \mathcal{X}_\ell^{\text{inf}} = \{(X, Y)\}_\ell$$

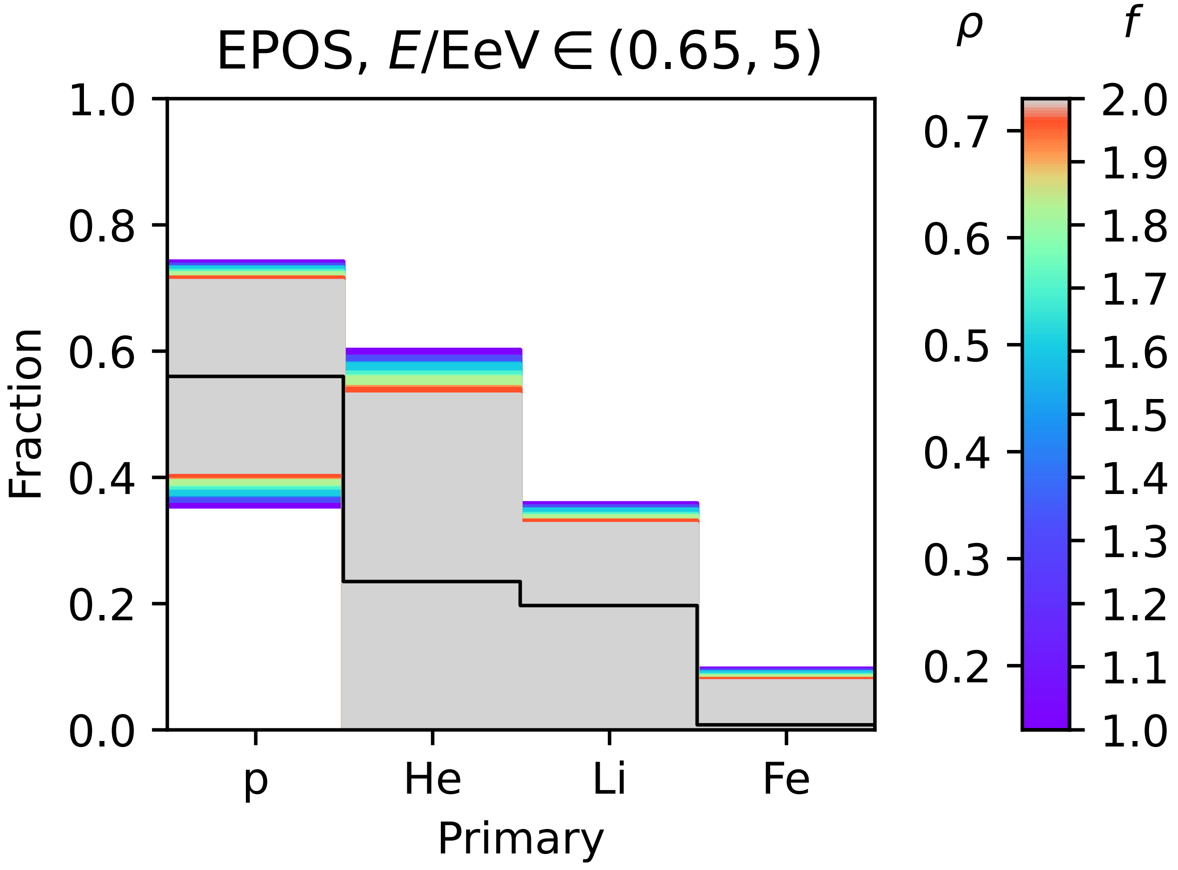
$$f \equiv \frac{\text{Var}(\bar{x})}{\text{Var}(\bar{X})}$$

Effective factor of included events





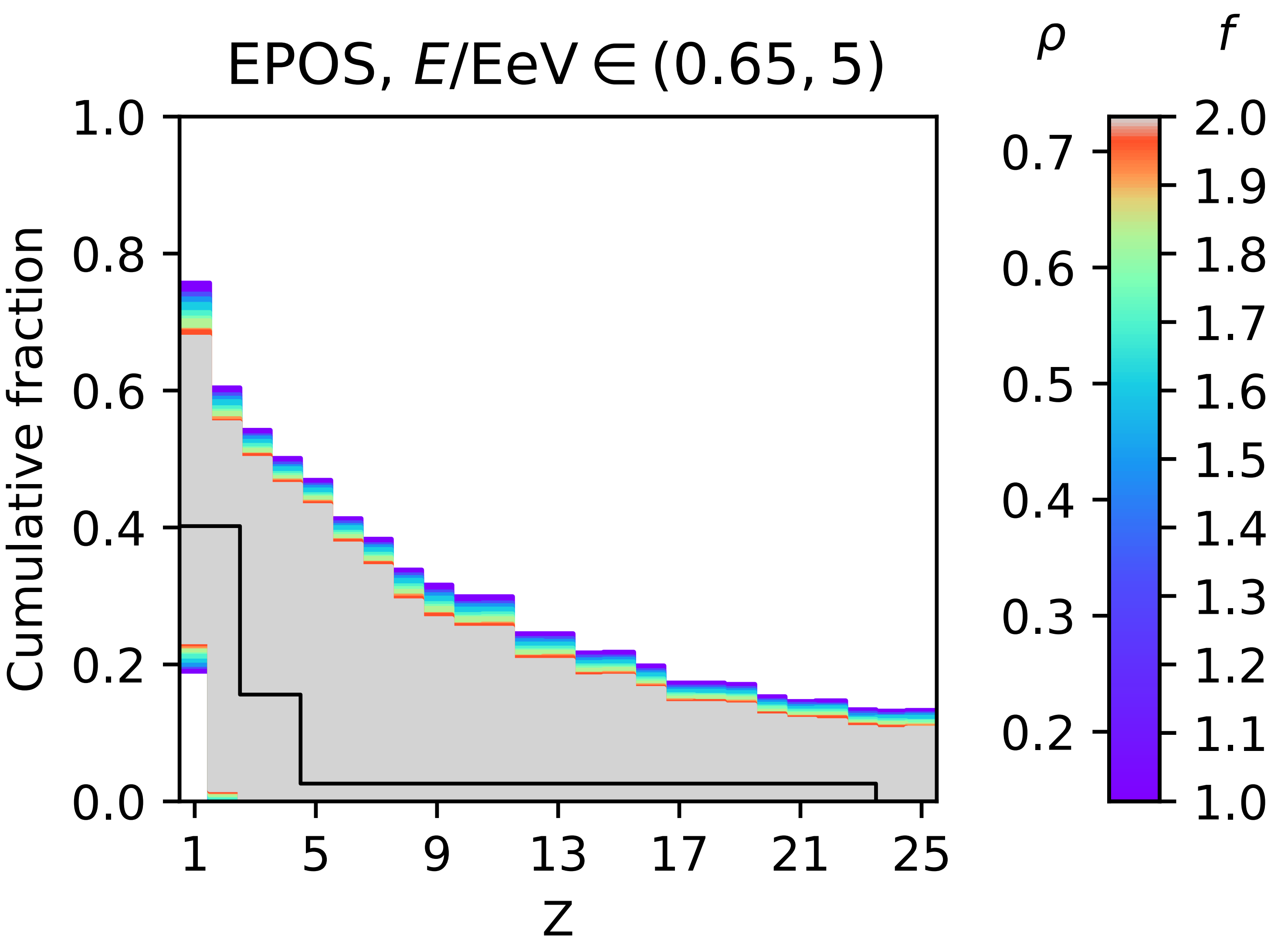
Including ground data



With ~70% correlation we can effectively double the statistical power

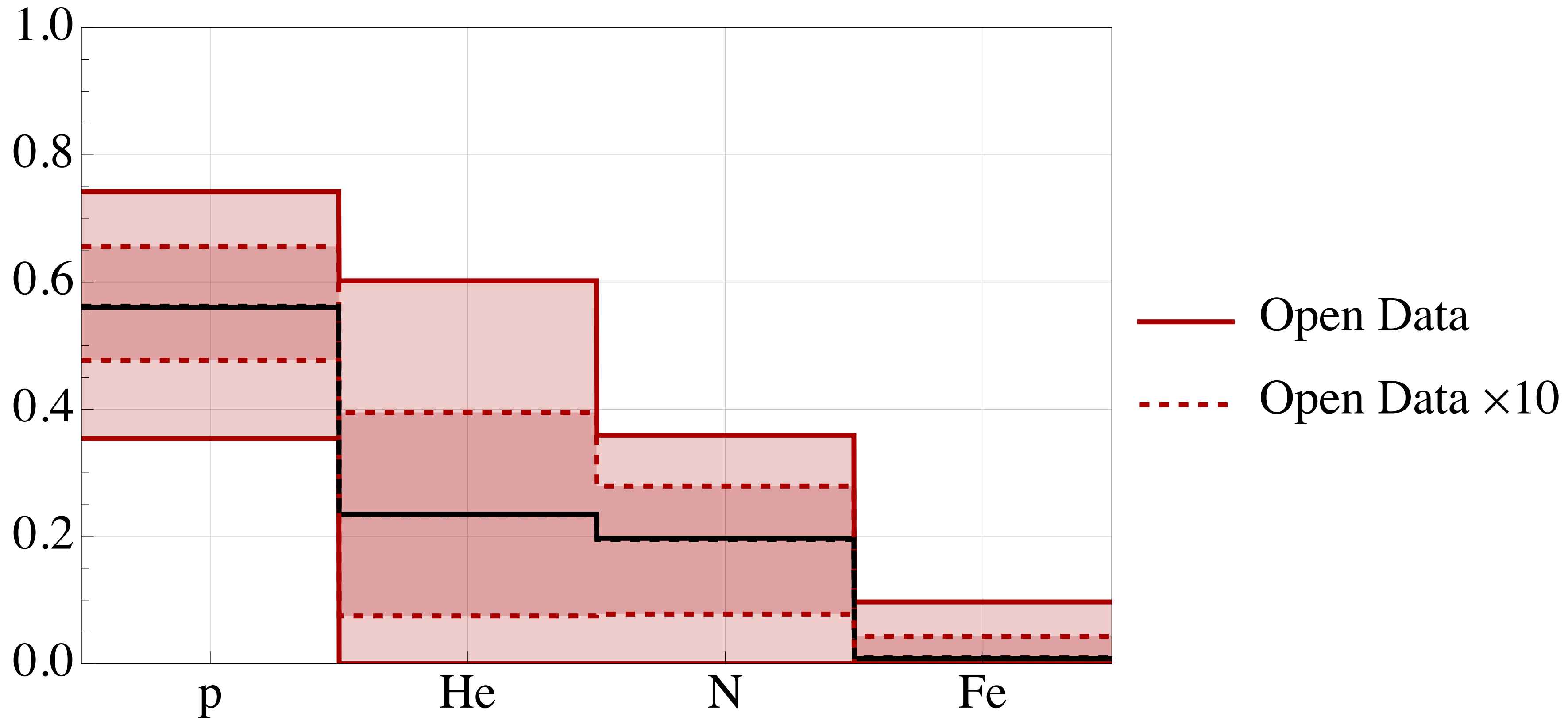
ρ~30% with (non)-linear fits
(Auger Coll.: 1710.07249)

ρ~70% with Deep Neural Network
(Auger Coll.: 2101.02946)



Including ground data

Projecting for larger datasets (but still a small fraction of the full)



Summary

- We build a simple framework for inference of UHECR mass composition

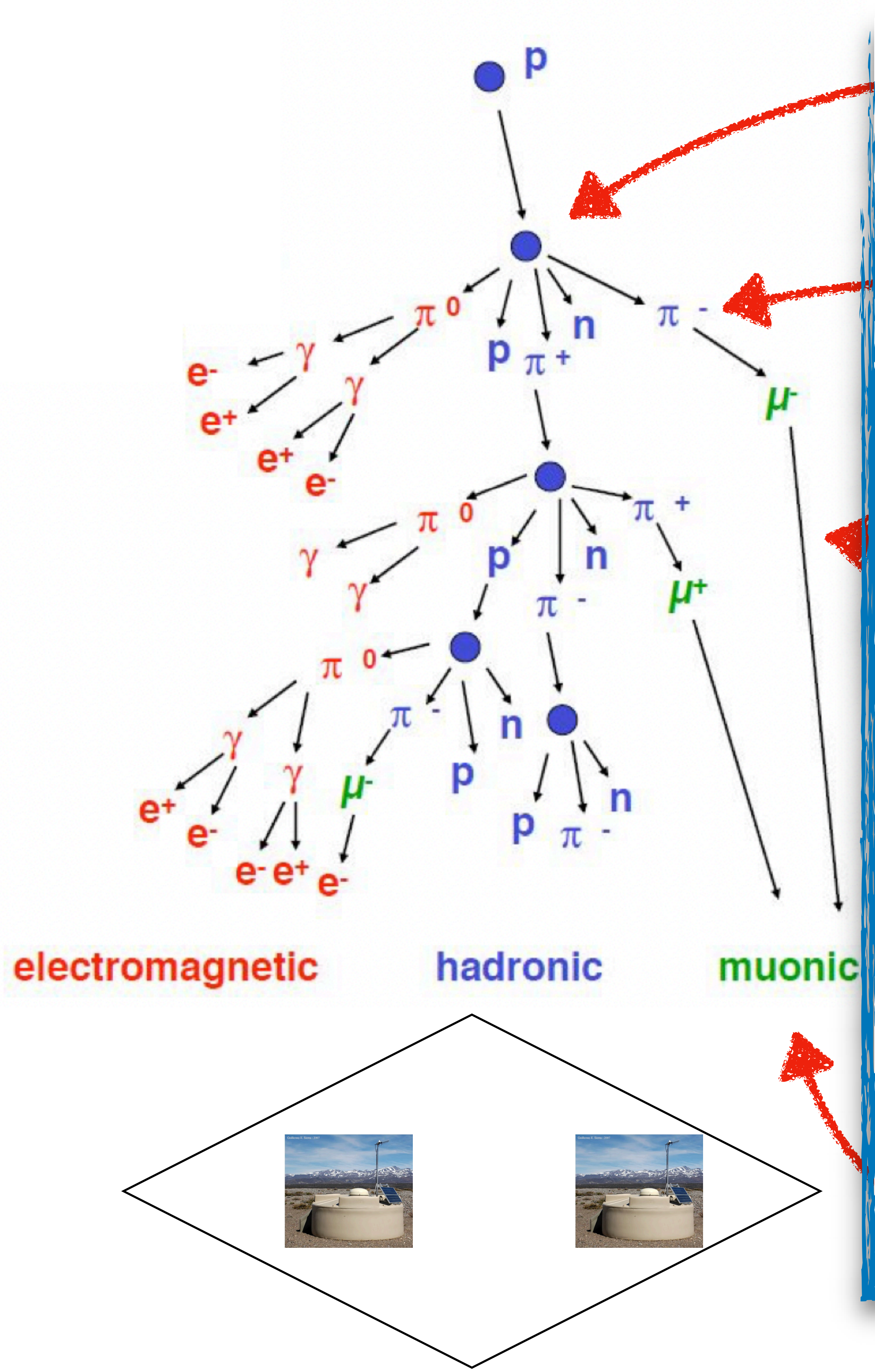
- n = 3 moments are good discriminating features

- Limited by very low statistics

- Many ideas for improvements

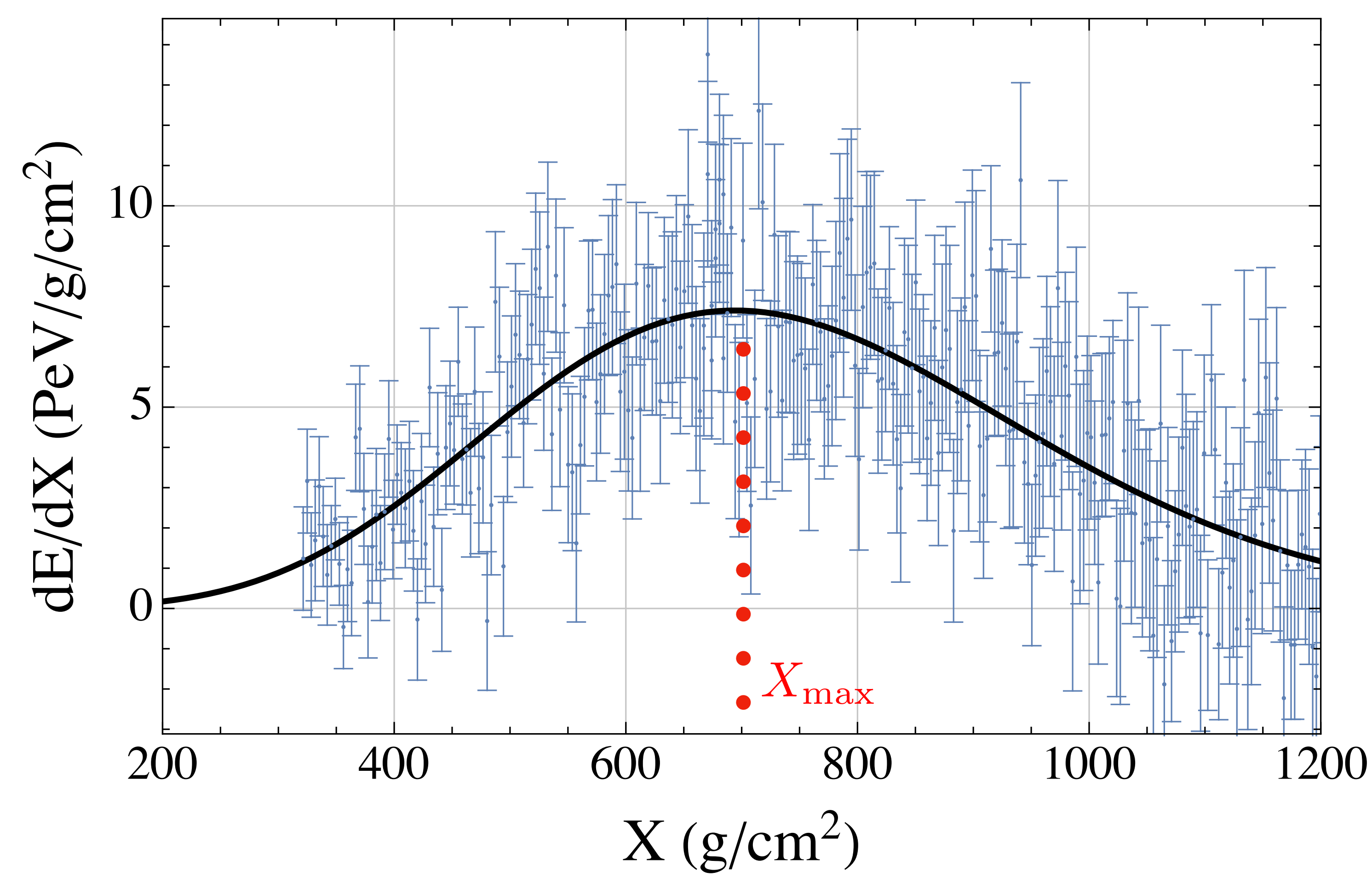
Backup slides

Extensive Air Showers (EAS)



Cherenkov detectors

$$X_{\rm ground} \sim 1200\ {\rm g/cm^2}$$



(Gaisser, Hillas, '77)

$$f_{GH}(X) = \left(\frac{{\rm d}E}{{\rm d}X}\right)_{\rm max} \left(\frac{X - X_0}{X_{\rm max} - X_0}\right)^{\frac{X_{\rm max} - X_0}{\lambda}} \exp\left(\frac{X_{\rm max} - X}{\lambda}\right)$$

Interlude: Bayesian inference

$\mathbf{X} = x_1, \ldots, x_n$ data sample $\qquad \theta$ parameters, $x \sim p(x|\theta)$

Likelihood

Prior distribution

$$P(\theta|\mathbf{X}) = \frac{P(\mathbf{X}|\theta)P(\theta)}{P(\mathbf{X})} = \frac{\mathcal{L}(\theta|\mathbf{X})P(\theta)}{\int \mathcal{L}(\theta|\mathbf{X})P(\theta)\mathrm{d}\theta}$$

Posterior distribution

Evidence

Maximum Likelihood Estimate (MLE): select θ that maxime $\mathcal{L}$

Maximum a Posteriori Estimation (MPE) for Bayesian people

Interlude: Bayesian inference

$\mathbf{X} = x_1, \ldots, x_n$ data sample

θ parameters, $x \sim p(x|\theta)$

$\mathbf{X} \equiv X_{\max}$

$\theta \equiv w = (w_{\mathrm{p}}, w_{\mathrm{He}}, \ldots, w_{\mathrm{Fe}}) \quad \sum_i w_i = 1$

$\mathcal{L}(\theta|\mathbf{X})$ from simulations

Likelihood

Prior distribution

$$P(\theta|\mathbf{X}) = \frac{P(\mathbf{X}|\theta)P(\theta)}{P(\mathbf{X})} = \frac{\mathcal{L}(\theta|\mathbf{X})P(\theta)}{\int \mathcal{L}(\theta|\mathbf{X})P(\theta)\mathrm{d}\theta}$$

Posterior distribution

Evidence

Maximum Likelihood Estimate (MLE): select θ that maxime $\mathcal{L}$

Maximum a Posteriori Estimation (MPE) for Bayesian people

Interlude: Bayesian inference

$\mathbf{X} = x_1, \ldots, x_n$ data sample $\qquad \theta$ parameters, $x \sim p(x|\theta)$

$\mathbf{X} \equiv X_{\max} \qquad \theta \equiv w = (w_{\mathrm{p}}, w_{\mathrm{He}}, \ldots, w_{\mathrm{Fe}}) \quad \sum_i w_i = 1$

Likelihood

Prior distribution

$$P(\theta|\mathbf{X}) = \frac{P(\mathbf{X}|\theta)P(\theta)}{P(\mathbf{X})} = \frac{\mathcal{L}(\theta|\mathbf{X})P(\theta)}{\int \mathcal{L}(\theta|\mathbf{X})P(\theta)\mathrm{d}\theta}$$

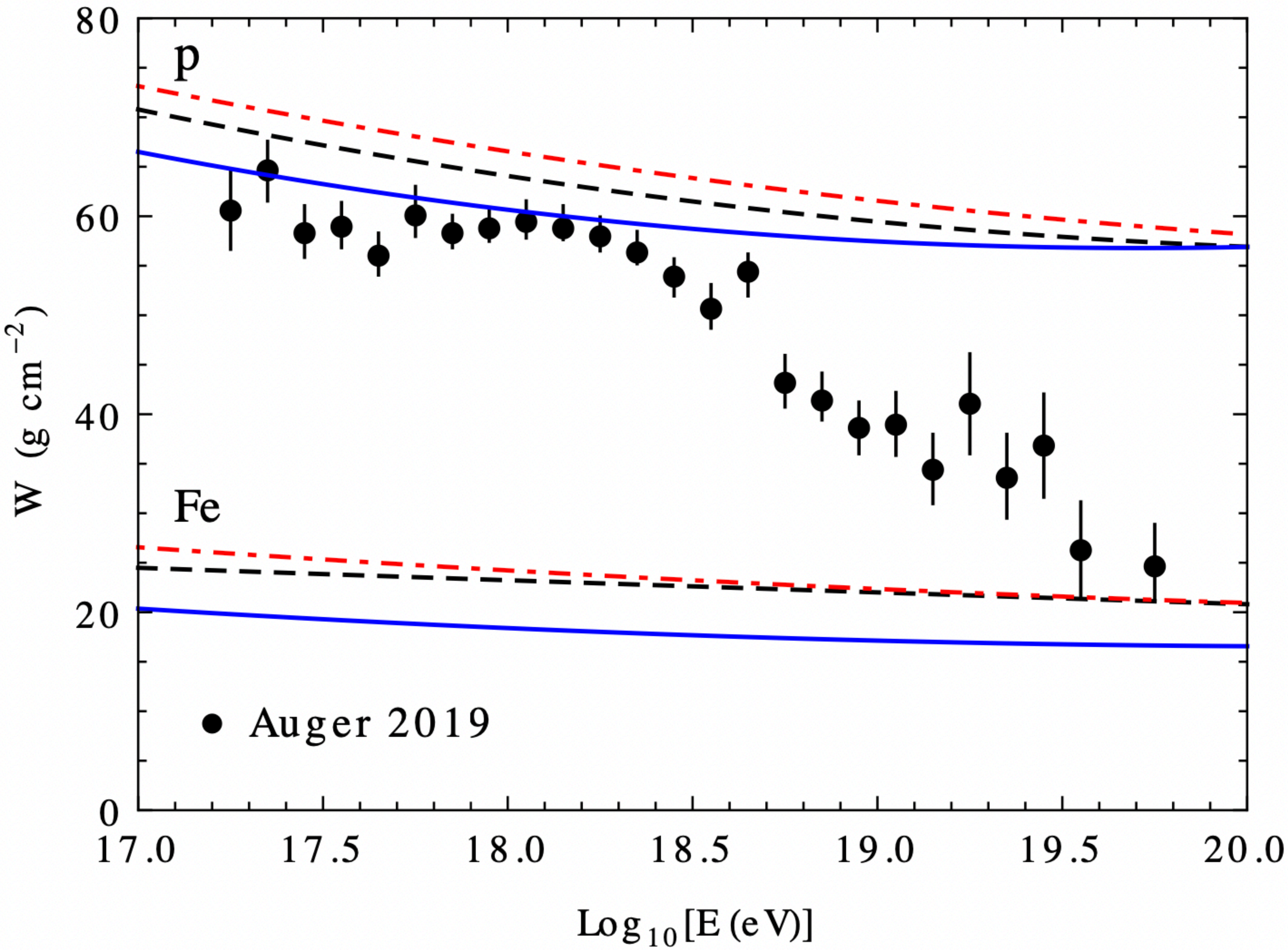
Posterior distribution

Evidence

Maximum Likelihood Estimate (MLE): select θ that maxime $\mathcal{L}$

Maximum a Posteriori Estimation (MPE) for Bayesian people

(Lipari: 2012.06861)



$$W = \left(\langle X_{\max}^2 \rangle - \langle X_{\max} \rangle^2\right)^{1/2}$$

Simulations perfomed with CORSIKA (https://www.iap.kit.edu/corsika/)

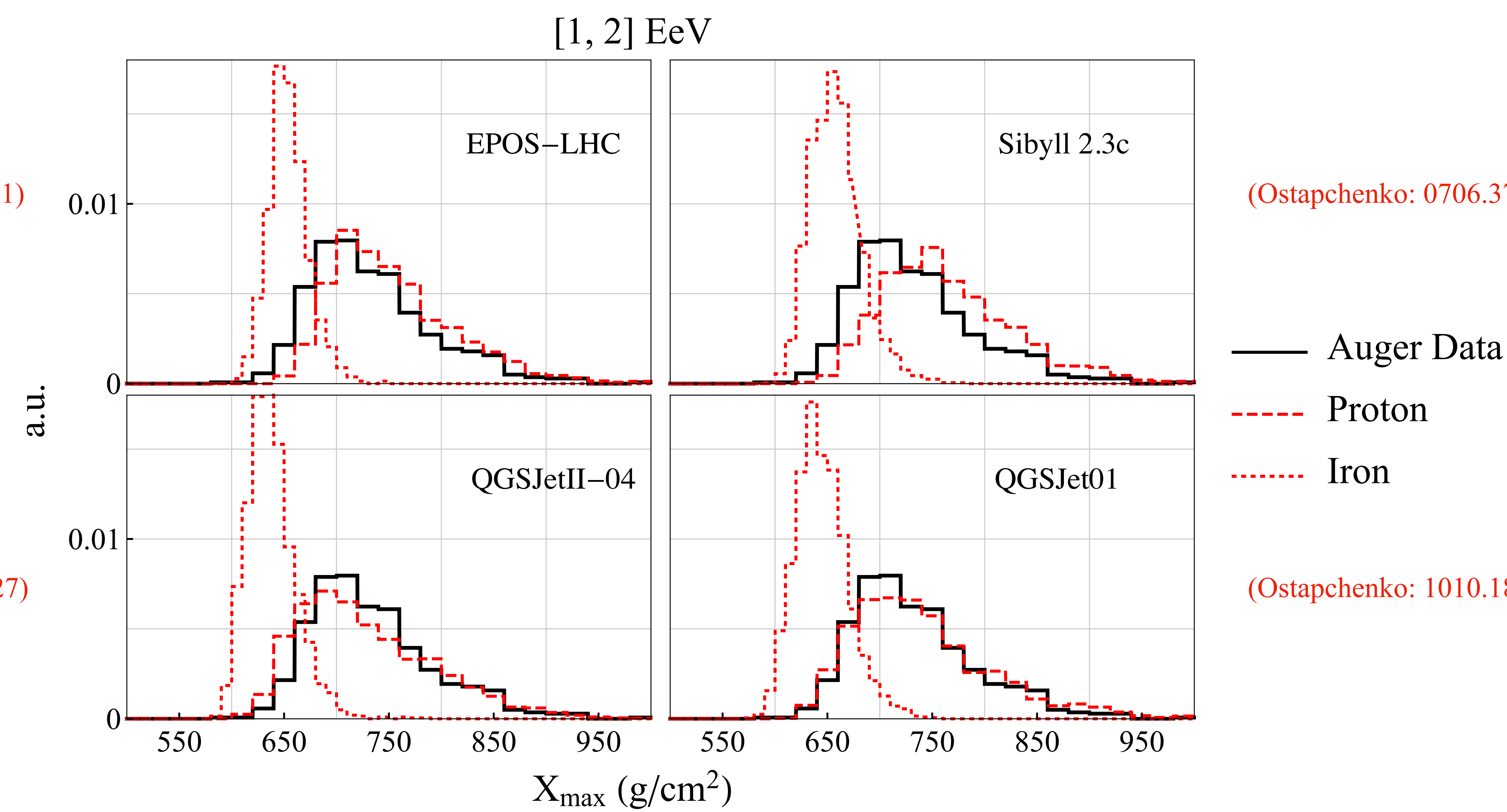
Single or two-element mixture cannot reproduce the data

(depending on the model…)

The results will be model-dependent

(Pierog et al: 1306.0121)

(Ostapchenko: 0706.3784)



(Riehn et al: 1709.07227)

(Ostapchenko: 1010.1869)

Data

Pierre Auger Open Data 2021

- $\sim 10\%$ of total data
- 1602 hybrid showers in $E \in [0.6, 60]$ EeV
- Split in three bins: $E \in [0.6, 1],\ [1, 2],\ [2, 5]$ EeV
- ~ 500 events per bin

$$P_{\text{Aug}}(X_{\text{max}}|E)$$

Simulations with CORSIKA

- 4 hadronic models
- 26 primaries from p to Fe
- 2000 shower per element/bin/model
- 624000 simulations

Bonus achievement: get complaints from both IJS and CERN clusters

$$P_{\text{sim}}(X_{\text{max}}|Z, E)$$

Convolute all with detector effects

(Auger Coll.: 1409.5083)

Probability Distribution Function (PDF)

Each data/simulation point is given as $(X_{\rm max}, \delta X_{\rm max})$

Data

$$P_{\rm Aug}(X_{\rm max}|E) = \frac{1}{N}\sum_{j=1}^{N} \mathcal{N}\left(X_{\rm max} \mid X^j_{\rm max}, \delta X^j_{\rm max}\right)$$

Simulations

Set of inputs

Detector effects

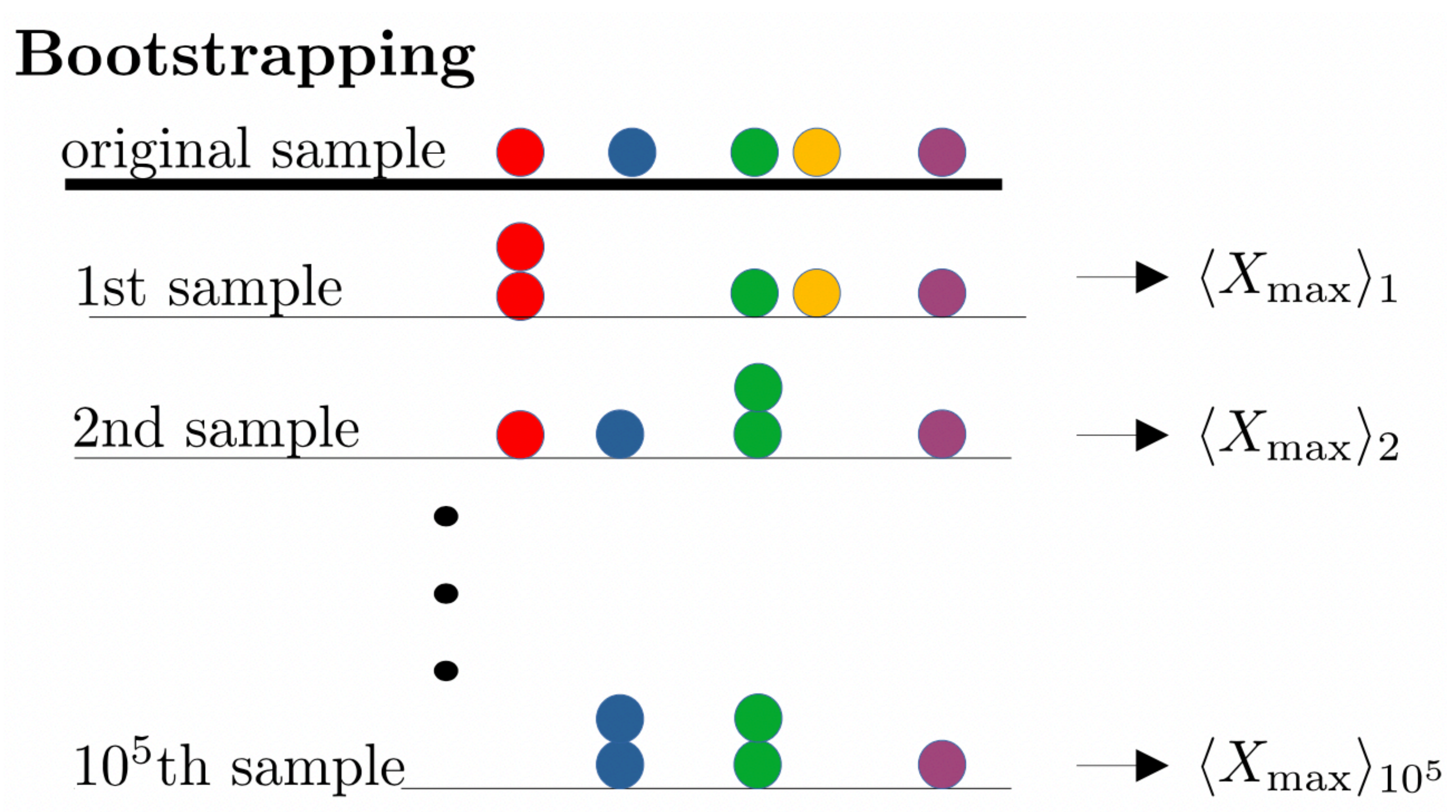
$$P_{\rm sim}(X_{\rm max} \mid S = \{E, Z, H\}) = \frac{1}{\tilde{N}}\sum_{j} \int {\rm d}\tilde{X}\, \mathcal{N}\left(\tilde{X} \mid X^j_{\rm max}, \delta X^j_{\rm max}\right) \times R(X_{\rm max} - \tilde{X}) \times \epsilon(\tilde{X})$$

Moments

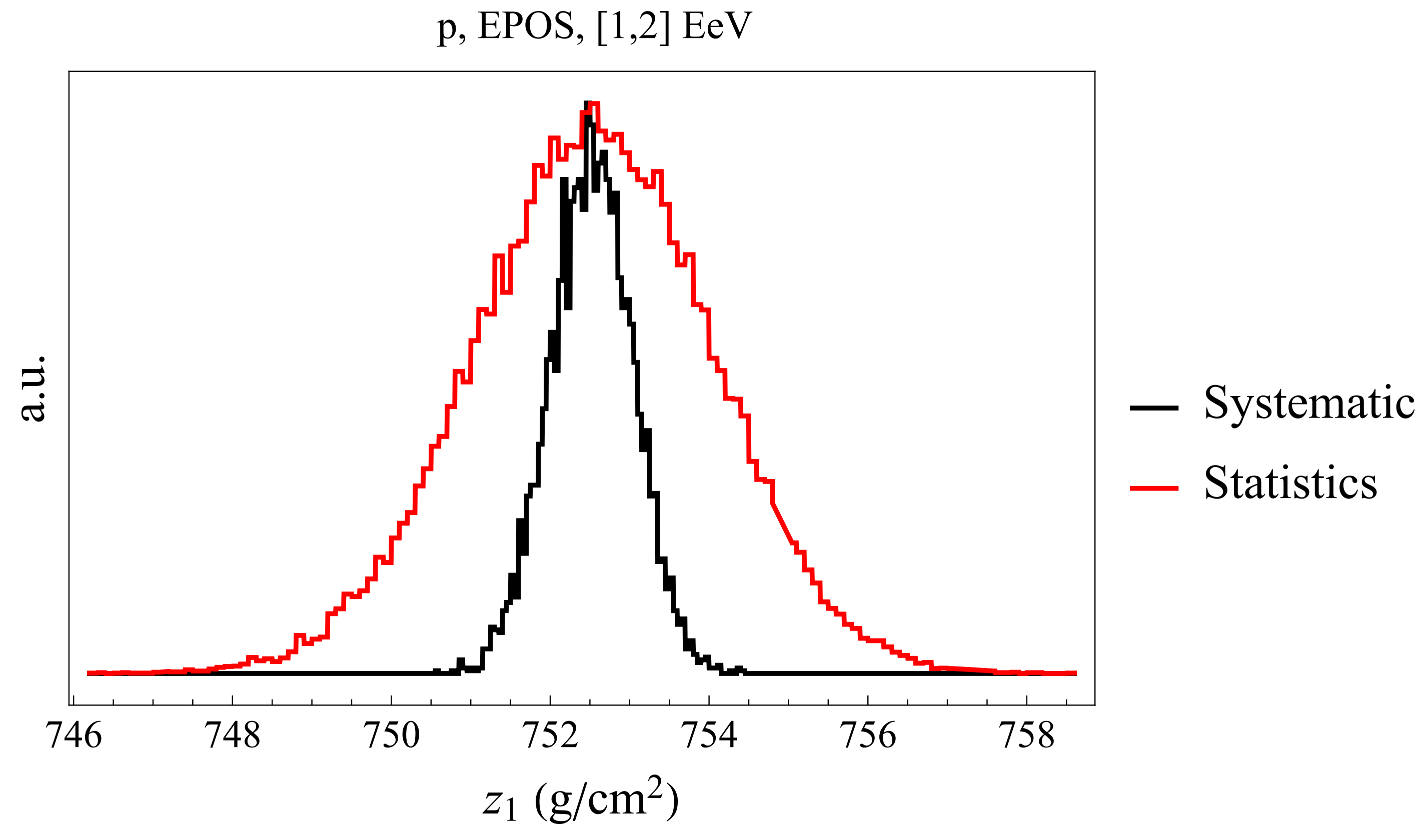
$$\langle X_{\max}^n \rangle_Z = \frac{\int P(X_{\max} \mid Z)\; X_{\max}^n \; \mathrm{d}X_{\max}}{\int P(X_{\max} \mid Z)\; \mathrm{d}X_{\max}}$$

$$\langle X_{\max}^n \rangle(w) = \frac{\sum_Z \langle X_{\max}^n \rangle_Z \; w_Z}{\sum_Z \; w_Z}$$

All systematic uncertainties are included

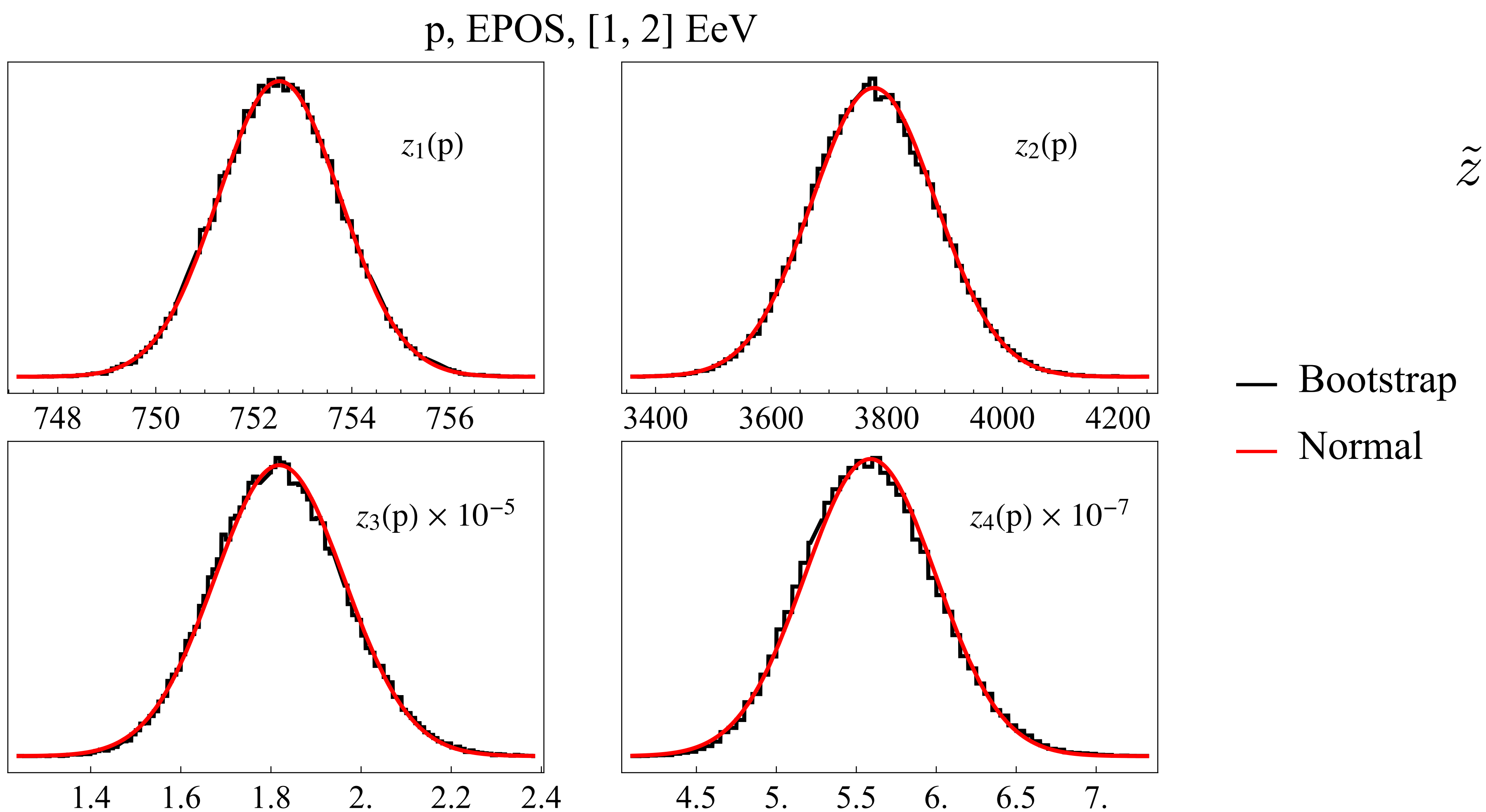


Include statistical uncertainties
(data and simulation)
by Bootstrapping



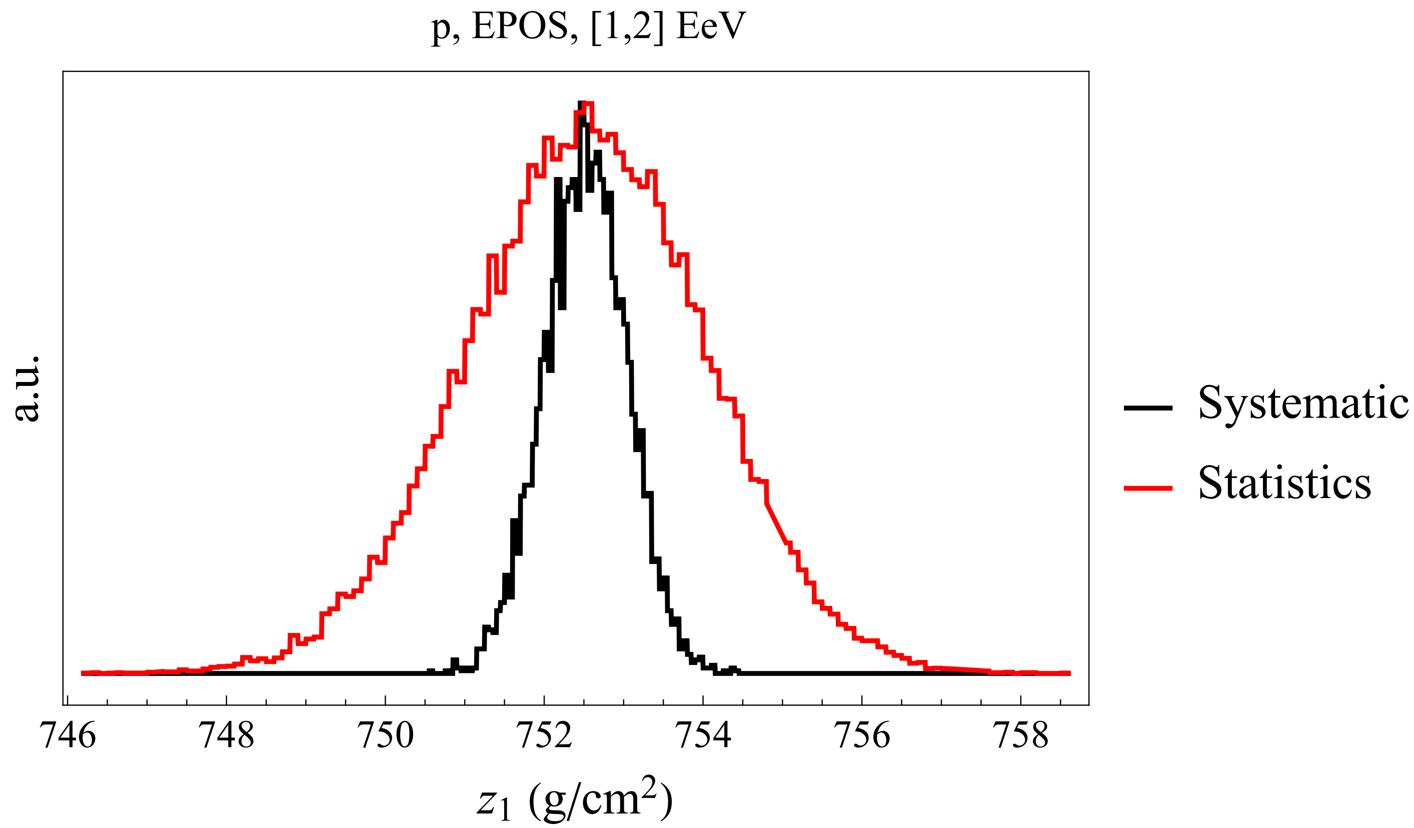
Simulations

$$z(w) \sim \mathcal{N}_n\Big(z \mid \mu(w),\ \Sigma(w)\Big)$$



Data

$$\tilde{z} \sim \mathcal{N}_n\left(z \mid \tilde{\mu}, \tilde{\Sigma}\right)$$

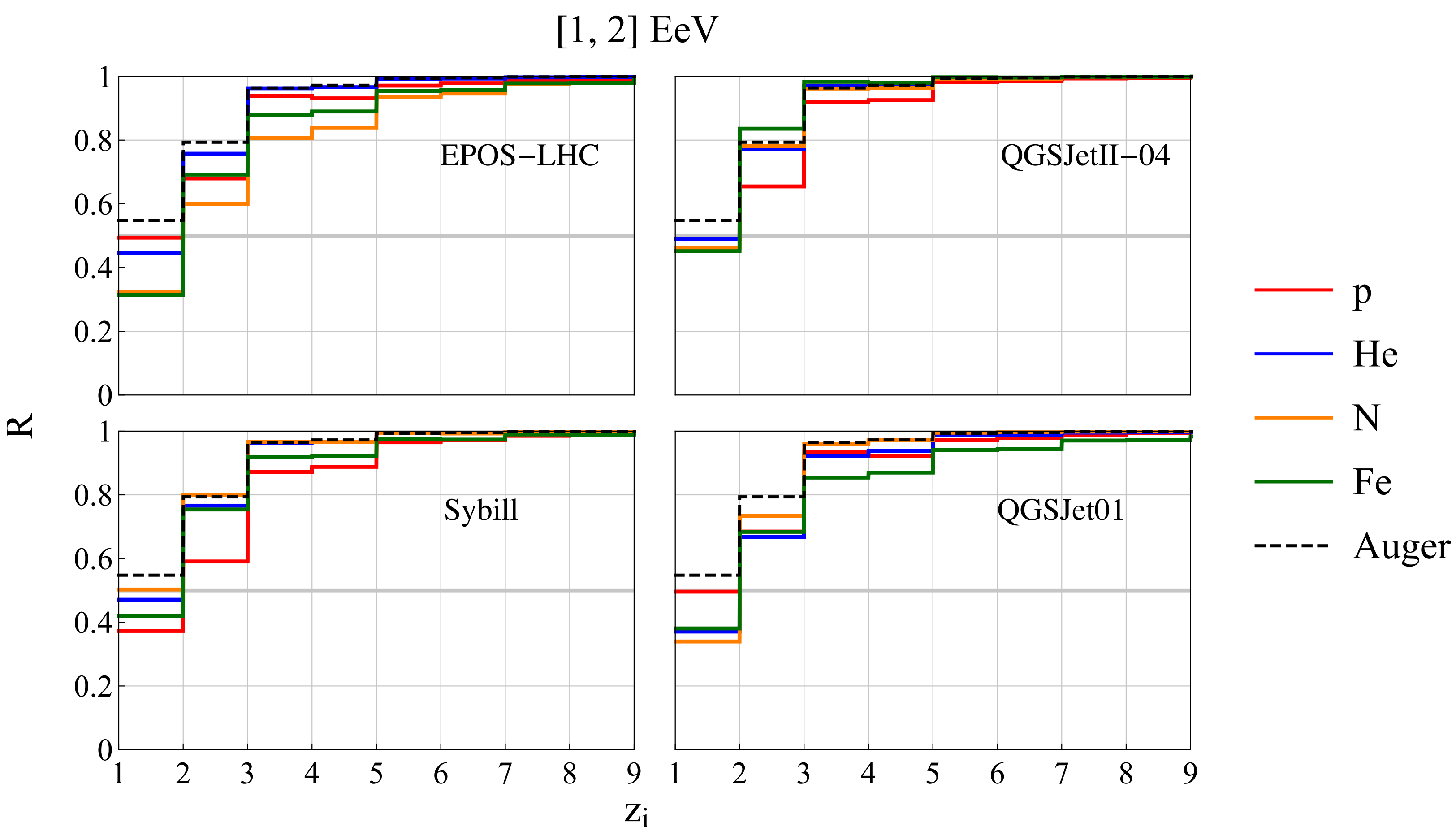


Simulations

$$z(w) \sim \mathcal{N}_n\Big(z \mid \mu(w),\ \Sigma(w)\Big)$$

Data

$$\tilde{z} \sim \mathcal{N}_n\left(z \mid \tilde{\mu}, \tilde{\Sigma}\right)$$



Higher moments are strongly correlated

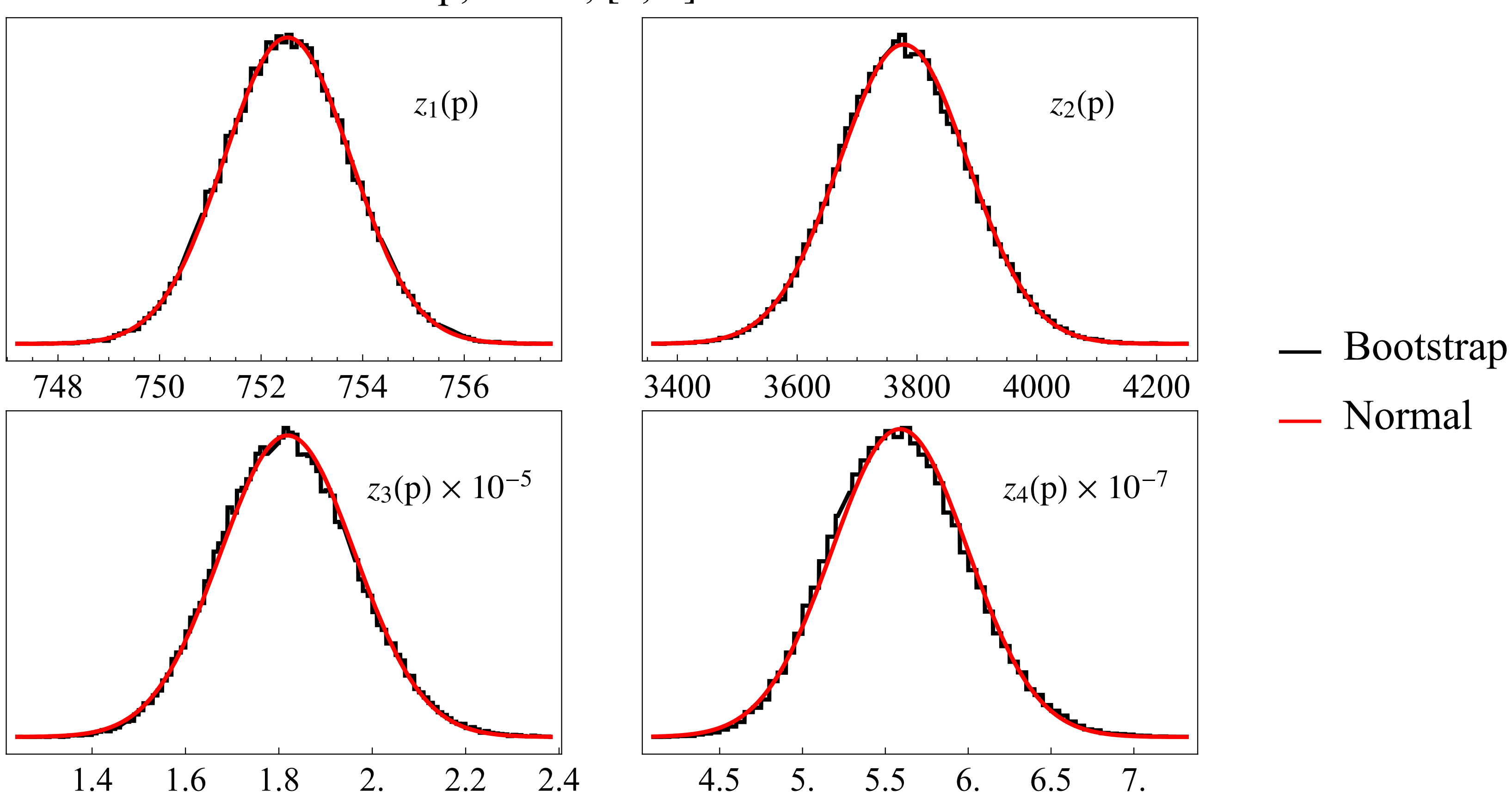
Moments

$$\langle X_{\max}^n \rangle_Z = \frac{\int P(X_{\max} \mid Z)\; X_{\max}^n \; \mathrm{d}X_{\max}}{\int P(X_{\max} \mid Z)\; \mathrm{d}X_{\max}}$$

$$\langle X_{\max}^n \rangle(w) = \frac{\sum_Z \langle X_{\max}^n \rangle_Z \; w_Z}{\sum_Z \; w_Z}$$

All systematic uncertainties are included

p, EPOS, [1, 2] EeV



Include statistical uncertainties (data and simulation) by Bootstrapping

Simulations

$$z(w) \sim \mathcal{N}_n\Big(z \mid \mu(w),\; \Sigma(w)\Big)$$

Data

$$\tilde{z} \sim \mathcal{N}_n\left(z \mid \tilde{\mu}, \tilde{\Sigma}\right)$$

Nested Sampling

Evidence $$Z = \int \mathcal{L}(w)\, \mathrm{Dir}(w)\, \mathrm{d}^D w = \int_0^1 \mathcal{L}(X)\, \mathrm{d}X\,,$$

- at step $k = 1$, sample N_{live} points (compositions)
- select w_1 with lowest likelihood L_1; w_1 is a dead point
- at step $k > 1$, sample a new live point w from prior, with constraint $\mathcal{L}(w) > L_{k-1}$
- Find the dead point w_k, with likelihood L_k
- calculate volume of prior region with $L_{k-1} < \mathcal{L}(w) \le L_k$
- calculate evidence shift $\delta Z_k = L_k \delta X_k$

Can be estimated with Beta distributions

Output is a set of w_k with weights $u_k = \delta Z_k / Z$

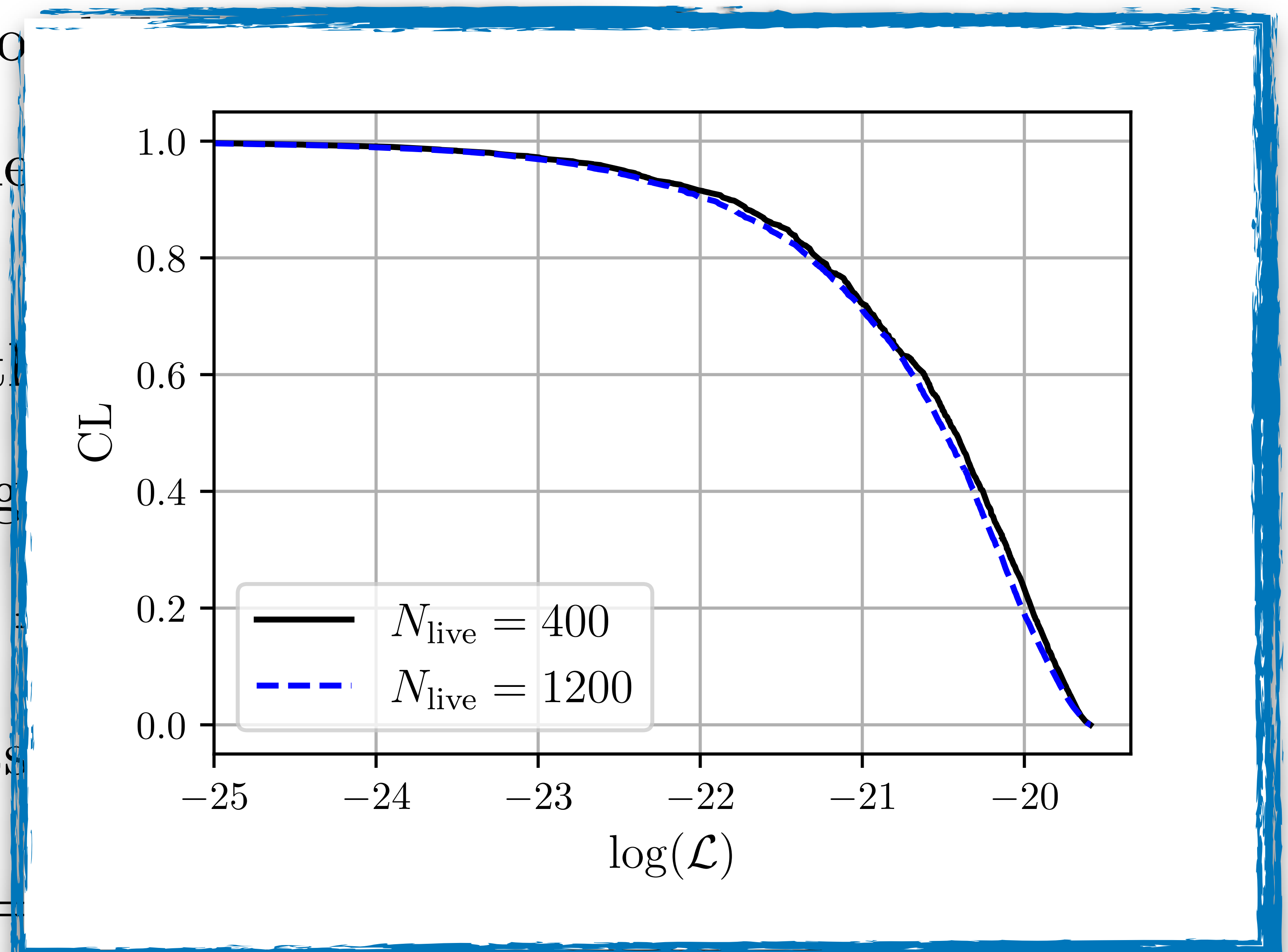
$$\mathrm{CL}(\mathcal{L}_0) = \sum_{(w_k, u_k)\ |\ \mathcal{L}(w) \ge \mathcal{L}_0} u_k$$

Nested Sampling

Evidence $$Z = \int \mathcal{L}(w)\ \mathrm{Dir}(w)\ \mathrm{d}^D w = \int_0^1 \mathcal{L}(X)\ \mathrm{d}X\,,$$

- at step $k=1$, sample N_{live} points (compositions)
- select w_1 with lowest likeliho
- at step $k>1$, sample a ne $\mathcal{L}(w) > L_{k-1}$
- Find the dead point w_k, wit
- calculate volume of prior reg
- calculate evidence shift δZ_k

Output is a set of w_k with weights



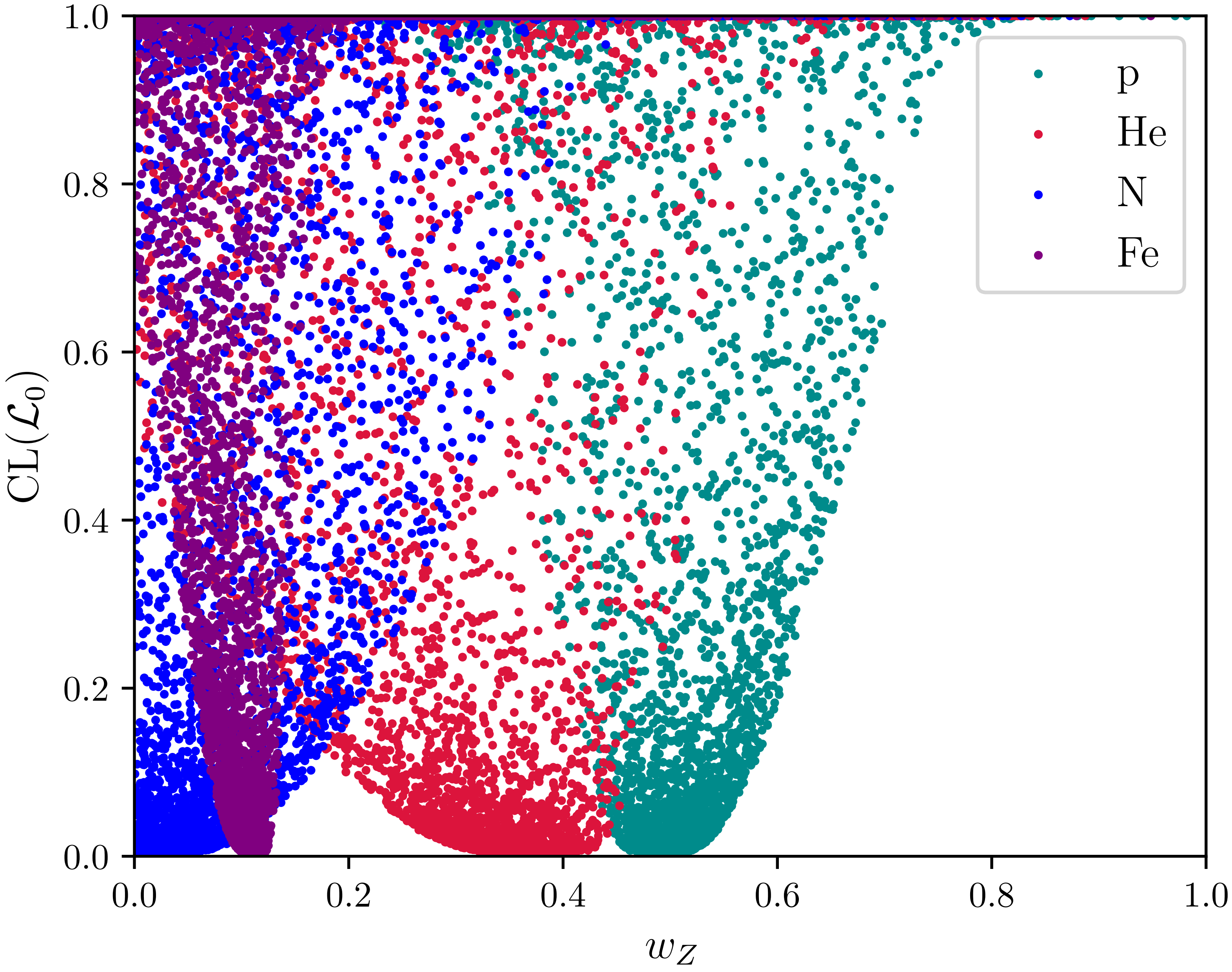
$$\mathrm{CL}(\mathcal{L}_0) =$$

$$(w_k,u_k)\ |\ \mathcal{L}(w)\geq\mathcal{L}_0$$

Full likelihood form

$$\mathcal{L} = (2\pi)^{-\frac{D}{2}} \det\left(\Sigma_w + \tilde{\Sigma}\right)^{-\frac{1}{2}} \exp\Big[-\frac{1}{2}\left(\mu_w^T \Sigma_w \mu_w + \tilde{\mu}^T \tilde{\Sigma} \tilde{\mu}\right) + \frac{1}{2}\left(\mu_w^T \Sigma_w^{-1} + \tilde{\mu}^T \tilde{\Sigma}^{-1}\right)\left(\Sigma_w^{-1} + \tilde{\Sigma}^{-1}\right)^{-1}\left(\Sigma_w \mu_w + \tilde{\Sigma}\tilde{\mu}\right)\Big]$$

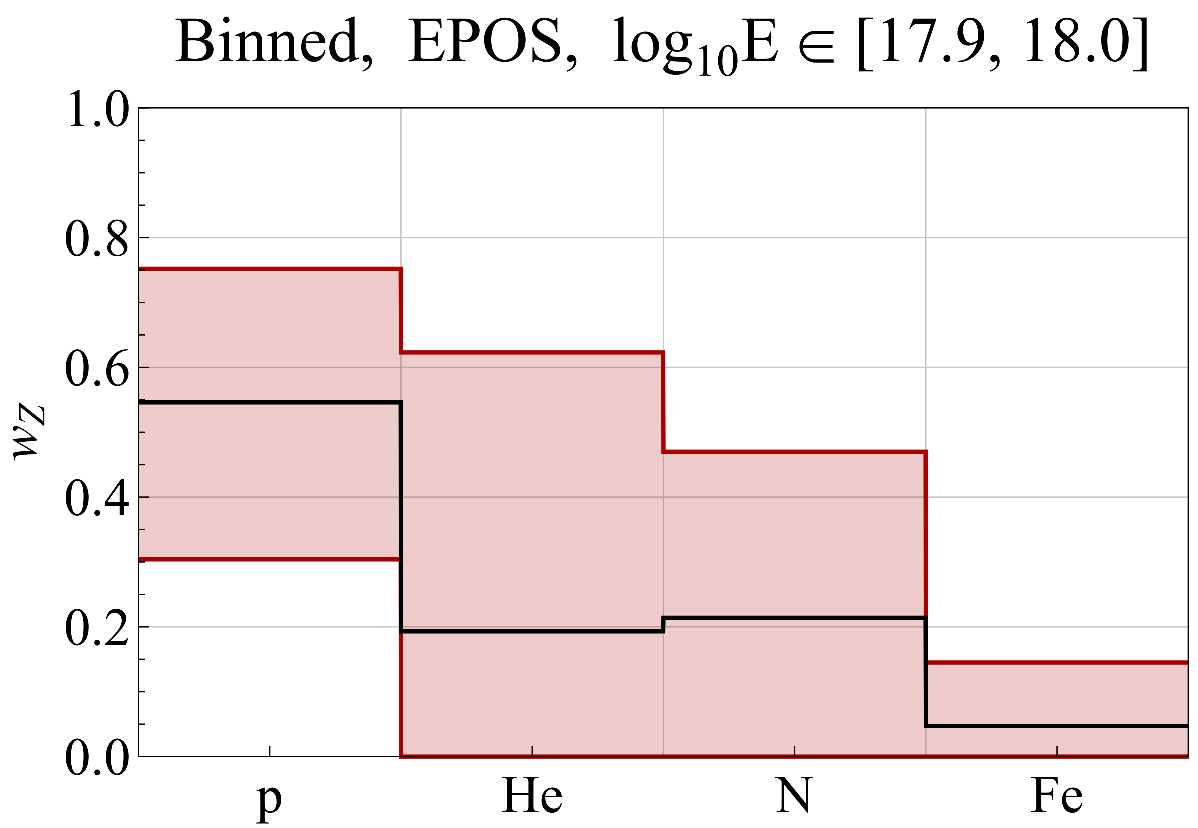
4 primary mixture

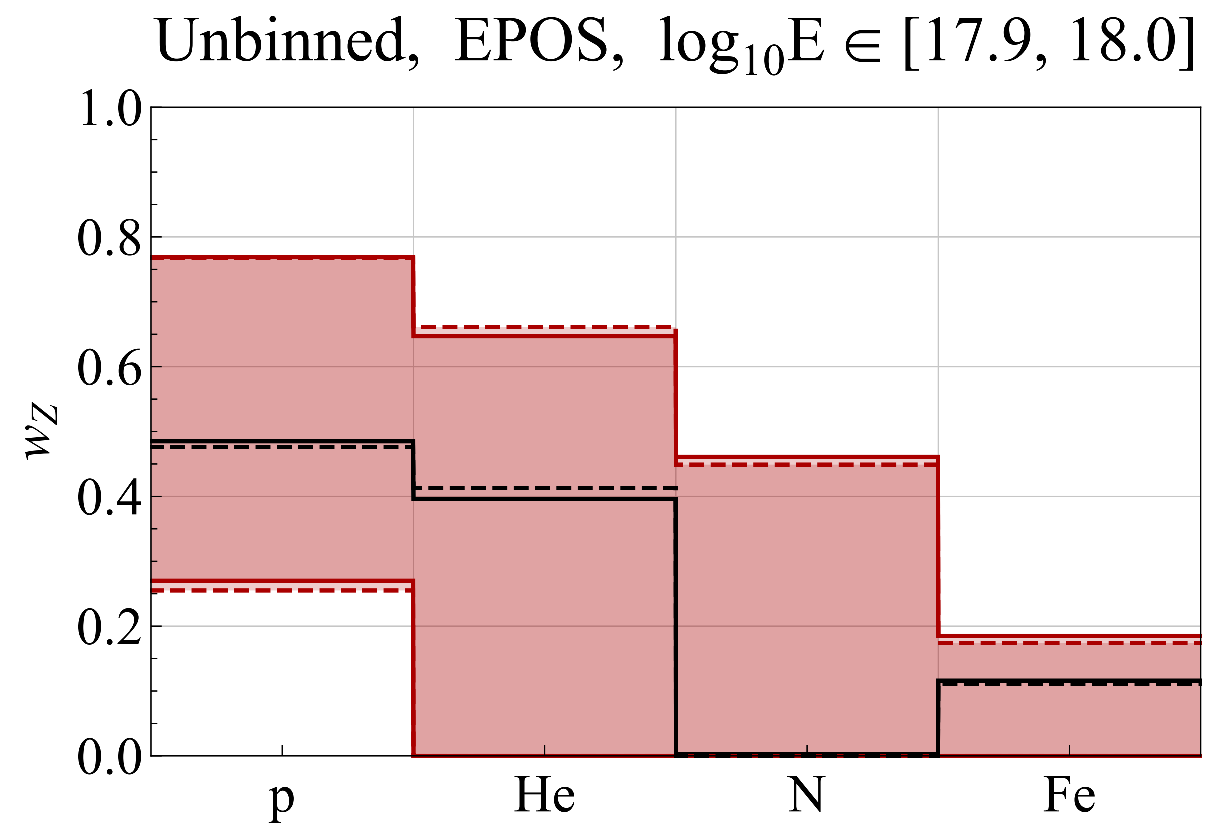


Projections of 4D log-likelihood

NS allows to efficiently sample the likelihood and find the Confidence Levels (CL)

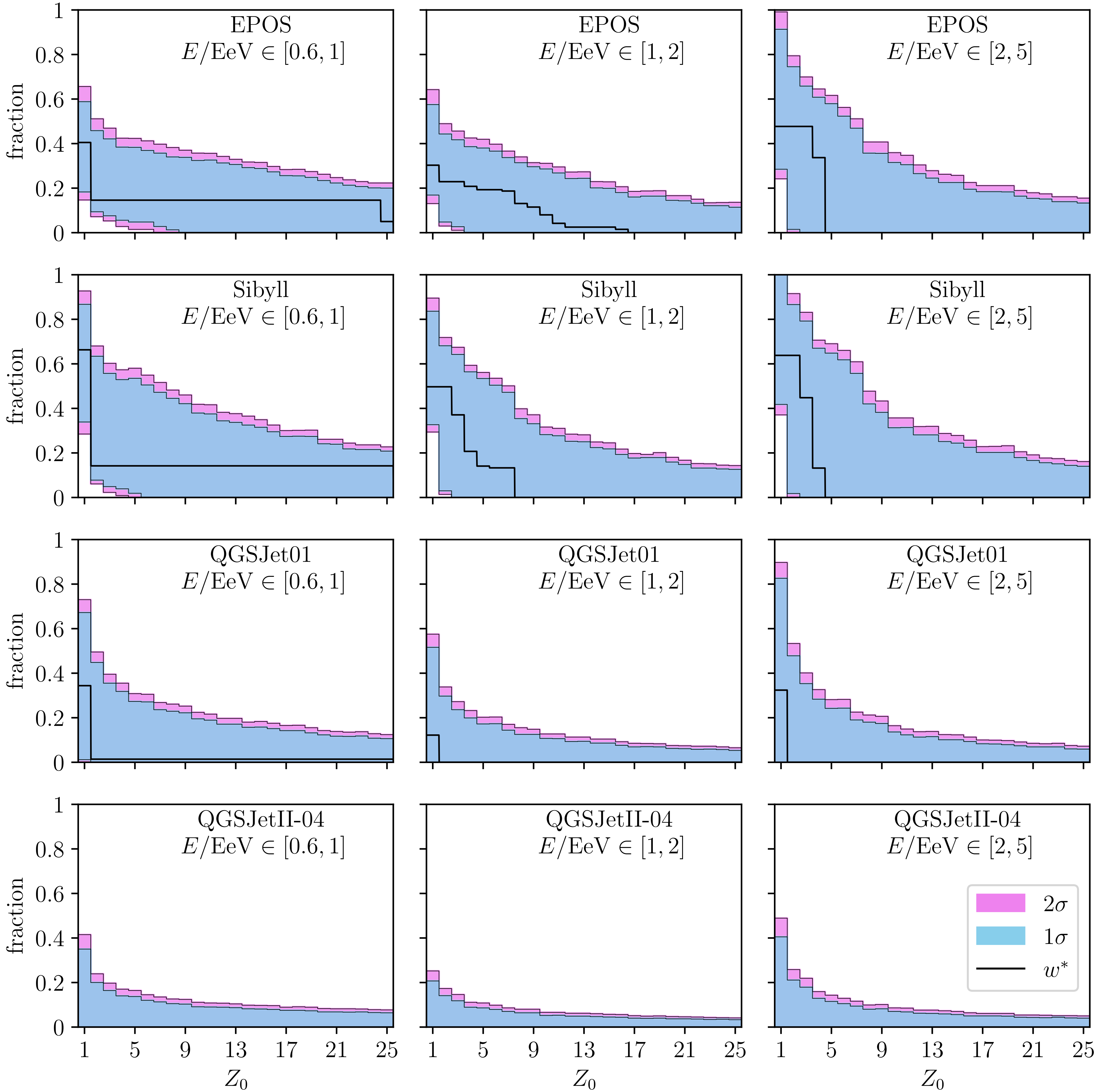
4 primary mixture





Consistent with results from 2001.02667

Results are unchanged increasing the number of features



EPOS and Sibyll (LHC-based) exclude 100% proton and can give bounds on heavy elements

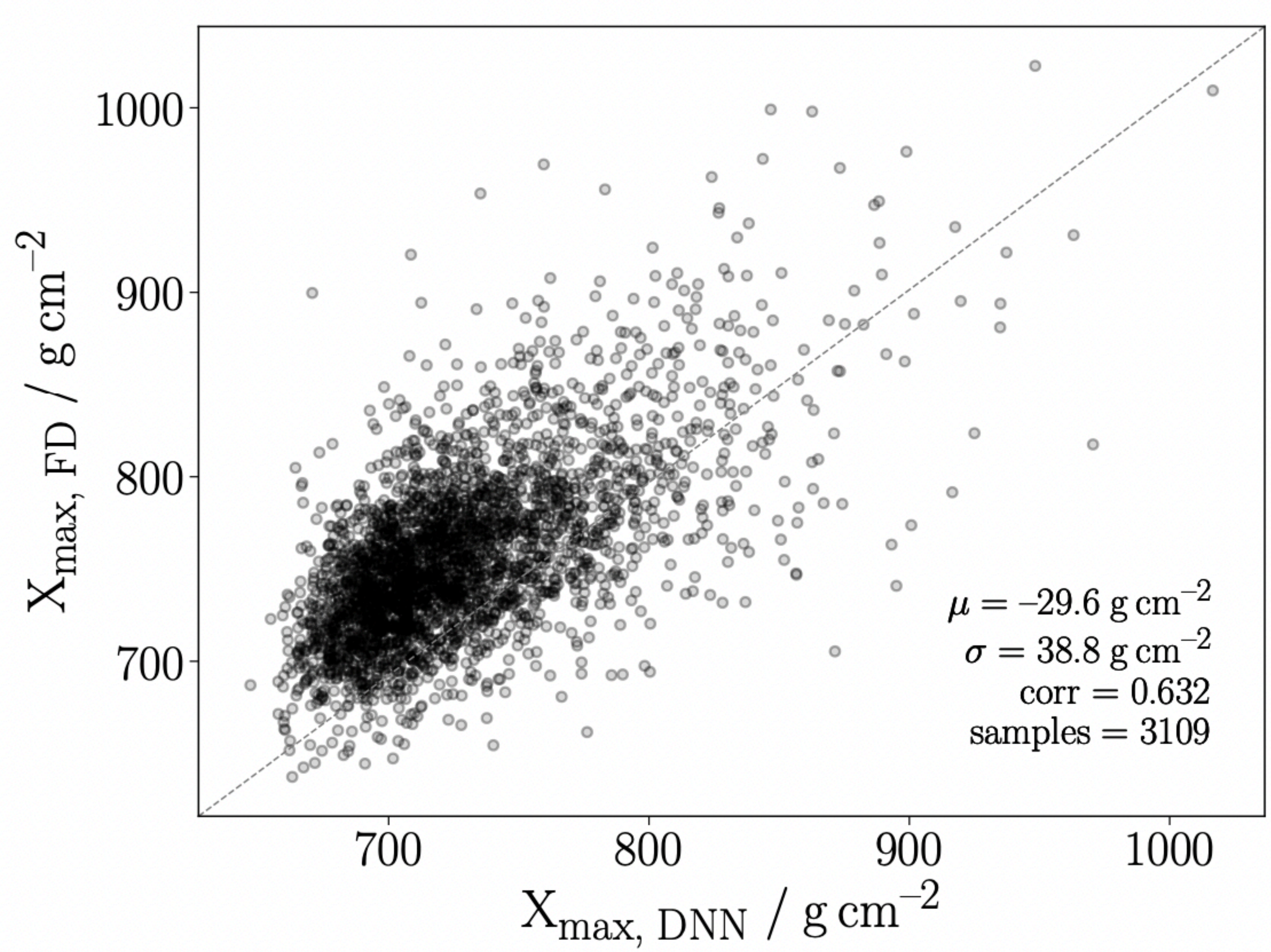
QGS models are consistent with 100% proton composition

SD to FD

Build a map from Surface Detector data to Fluorescent Detector data ($X_{\max}$)

Train Deep Neural Network (DNN) on simulated data

(Auger Coll.: 2101.02946)

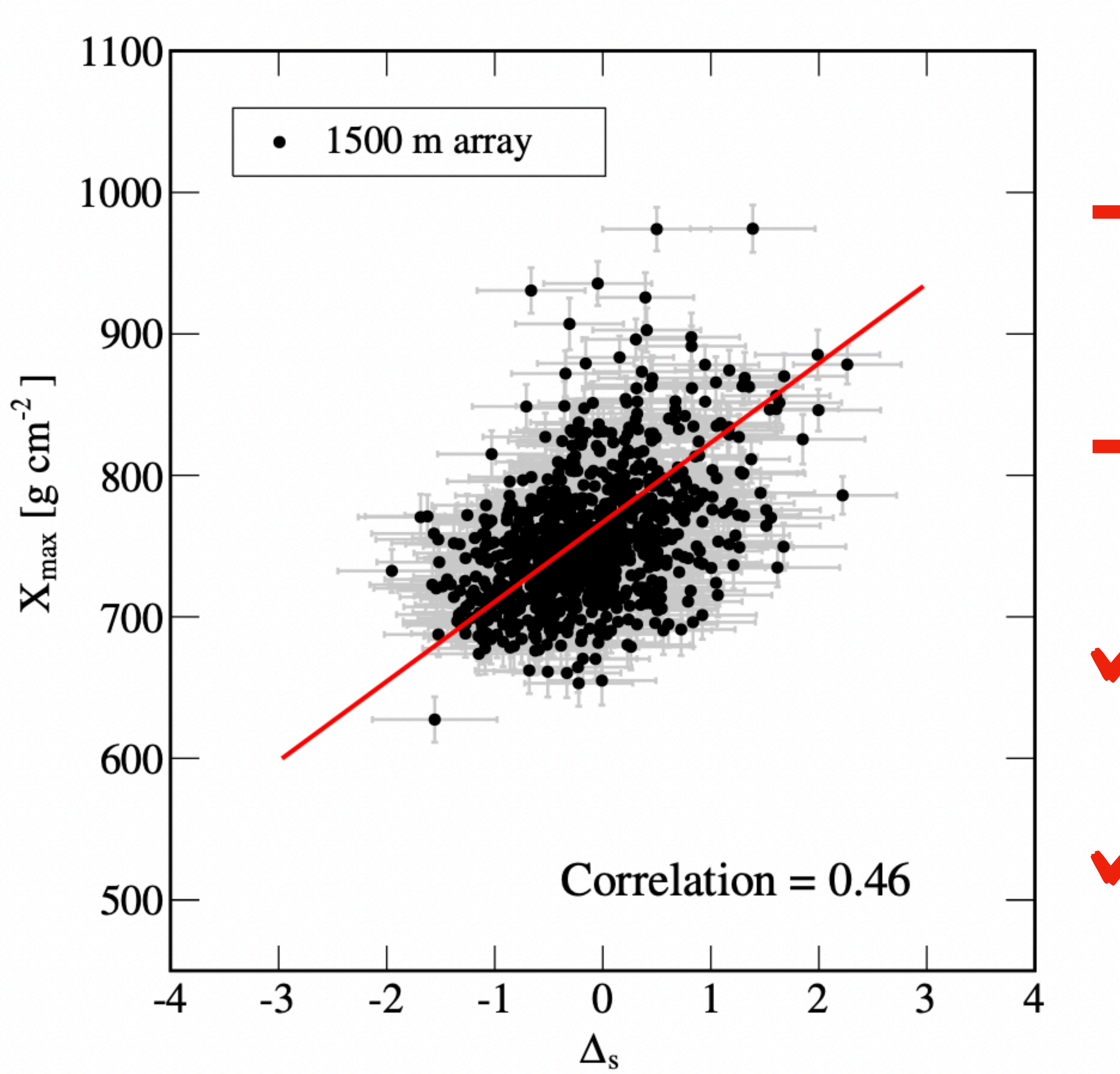


- ✓ Shows strong correlations
- – As good as simulations (ground data sims are ~wrong)
- – Trained with 4 primaries
- ✗ We do not have access to this DNN

SD to FD

Build a map from Surface Detector data to Fluorescent Detector data ($X_{\max}$)

(Auger Coll.: 1710.07249)



Build simple observable that correlates with FD

– Weaker correlation and large uncertainties

– Some quantities are given fits

✓ We can reproduce and use this

✓ We can (try to) improve on this