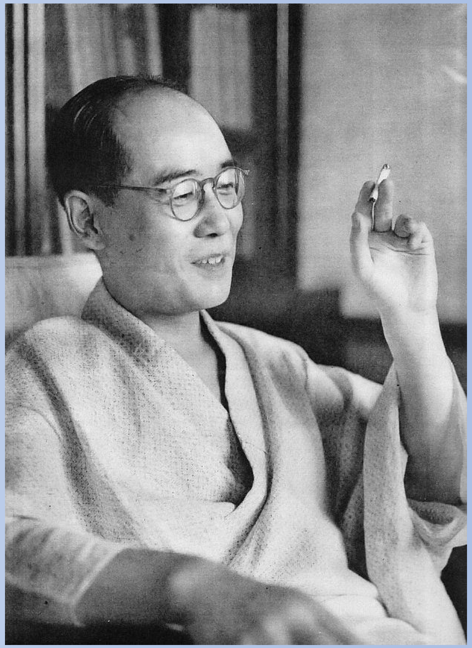
朝永振一郎



湯川秀樹

Exchange interactions between Europe and Japan in the 1930s: Tomonaga, Yukawa and nuclear theory

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Outline

- Exchange interactions in quantum mechanics and their applications
- Heisenberg, Fermi and the birth of theoretical nuclear physics
- Yukawa and the meson
- Tomonaga’s contribution
- Conclusions

Els (*Austausch*) in atoms

In 1926, Heisenberg introduced the idea of exchange interaction while studying the quantum mechanical description of a system of identical particles. He then successfully applied it to the spectrum of the He atom.

W. Heisenberg, Z. Phys. 1926, 1927, 1928

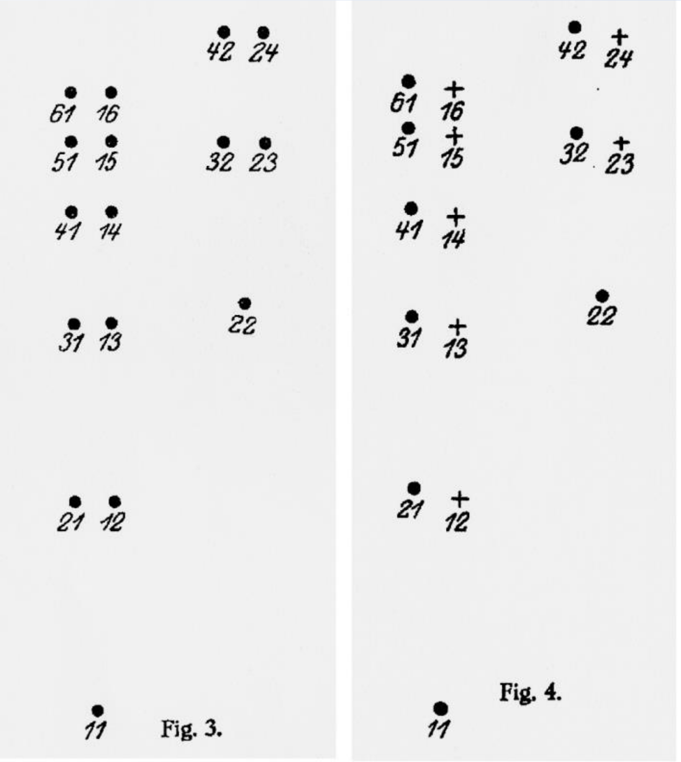


Fig. 3. Fig. 4.

$$\Delta E_{ab\pm} = \frac{1}{4\pi\epsilon_0}\int d^3r_1 d^3r_2 \, \frac{|\Psi_{ab\pm}(\mathbf{r}_1,\mathbf{r}_2)|^2}{|\mathbf{r}_1-\mathbf{r}_2|} = J_{ab} \pm K_{ab}$$

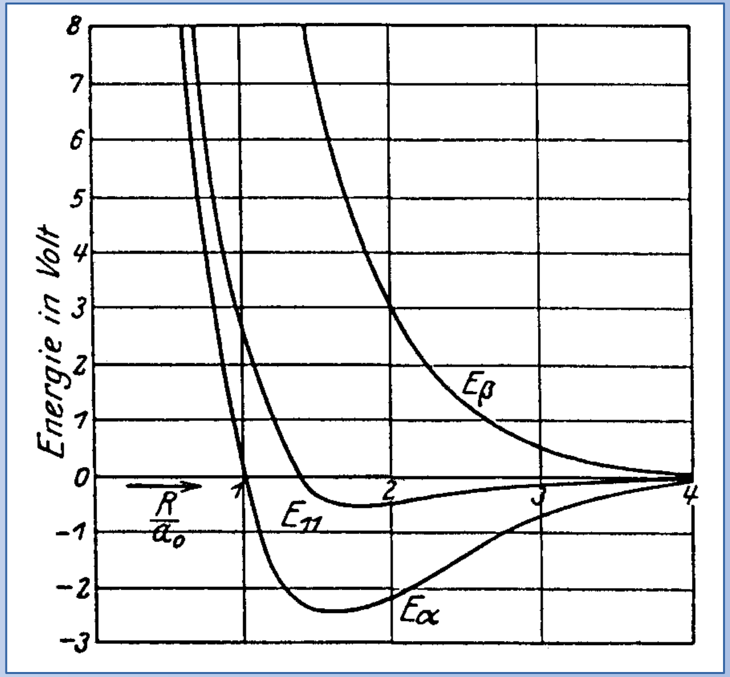
$$J_{ab} = \frac{1}{4\pi\epsilon_0}\int d^3r_1 d^3r_2 \, \frac{|\psi_a(\mathbf{r}_1)\psi_b(\mathbf{r}_2)|^2}{|\mathbf{r}_1-\mathbf{r}_2|}$$

$$K_{ab} = \frac{1}{4\pi\epsilon_0}\int d^3r_1 d^3r_2 \, \frac{\psi_a^\star(\mathbf{r}_1)\psi_b^\star(\mathbf{r}_2)\psi_a(\mathbf{r}_2)\psi_b(\mathbf{r}_1)}{|\mathbf{r}_1-\mathbf{r}_2|}$$

D. Tong – Lecture notes on Topics in QM

EIs (*Austausch*) molecules

In 1927, W. Heitler and F. London brought the idea of exchange interaction into molecular physics, and laid down the foundations of the quantum theory of the homeopolar chemical bond. Here, an intuitive notion of electrons literally exchanging their places around the different nuclei was briefly considered.



W. Heitler, F. London, Z. Phys. 1927

EIs (*Platzwechsel)* in nuclei

Up to 1932, the leading idea was that nuclei were made of protons and electrons, and that quantum mechanics somewhat failed in describing nuclear electrons. After the discovery of the neutron, Heisenberg assumed that it was a nuclear constituent. He modeled the p-n interaction after the molecular ion H_2^+. In his approach, it is *as if* the interaction is due to the exchange of an electron between two nucleons.

'If one brings a neutron and a proton to within a distance comparable to the dimensions of the nucleus, then – in analogy with the H_2^+ ion – a change of place of the negative charge will occur with a frequency given by a function (1/ ħ) J(r) [...]. The quantity J(r) corresponds to the Austausch- *or more correctly the* Platzwechsel-*integral of the molecular theory. One can illustrate this change of place again with the picture of electrons that have no spin and obey Bose statistics. But it is probably more correct to regard the exchange integral J(r) as a fundamental property of the proton-neutron pair, without wanting to reduce it to motion of electrons'*

(Heisenberg 1932, p.2, transl. In Carson 1996 p. 104) *C. Carson, Stud. Hist. Phil. Mod. Phys. 1996*

Notice the careful change of wording, due to the fact that there is only one electron in the H_2^+ ion. Hence, this is *not* an exchange interaction in the sense discussed above. However, if we forget about the electron, we can view the effect as due to proton-neutron exchange (*Austausch* again).

EIs (*Platzwechsel)* in nuclei

Heisenberg wrote down the Hamiltonian:

$$H = \frac{1}{2M}\sum_k \mathrm{p}_k^2 - \frac{1}{2}\sum_{k>l} J(r_{kl})(\rho_k^\xi \rho_l^\xi + \rho_k^\eta \rho_l^\eta) - \frac{1}{4}\sum_{k>l} K(r_{kl})(1+\rho_k^\zeta)(1+\rho_l^\zeta)$$

$$+ \frac{1}{4}\sum_{k>l}\frac{e^2}{r_{kl}}(1-\rho_k^\zeta)(1-\rho_l^\zeta) - \frac{1}{2}D\sum_k(1+\rho_k^\zeta),$$

$$\rho^\xi = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix},\ \rho^\eta = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix},\ \rho^\zeta = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

$\rho^\zeta = +1$ neutron

$\rho^\zeta = -1$ proton

The exchange integral *J(r)* is unspecified, apart from the requirement that it produces a short-range interaction. An additional interaction is assumed between neutrons, while only electrostatic repulsion acts between protons.

Fermi and β-decay

Between the end of 1933 and the beginning of 1934, Fermi solved both the problems of electrons in nuclei and of the spectrum of β-decay. Fermi's theory relies on the analogy with the emission of a photon by a decaying atom and puts together Pauli's neutrino hypothesis, Heinseberg's isobaric spin framework and second quantization. The analogy with the electromagnetic case implies the possibility both for electrons and neutrinos to be created ad annihilated.

E. Fermi, Ric. Sci.1933; Nuovo Cim. 1934; Z. Phys 1934

Immediately after the success of Fermi's theory, various people tried to unify β-radioactivity and the nuclear force by looking at the latter as due to an exchange of an electron and a neutrino (the pair thus would be Heisenberg's 'Bose electron'). It soon appeared that, due to the weakness of the Fermi interaction, the resulting nuclear force was way too weak. However, this was the first time that an analogy was drawn between electromagnetism and the nuclear force.

Heisenberg, letter 341 to Pauli, 18 January 1934

I. Tamm, Nature, 1934; D. Ivanenko, Nature, 1934

Quantum physics in Japan in the early XX century

1921. Nishina Yoshio (仁科 芳雄) goes to Europe and stays there until 1928, thus witnessing the unfolding of the quantum revolutions and greatly contributing himself. Back in Japan, he starts theoretical physics there.

1929. Heisenberg and Dirac, invited by Nishina, visit Japan and give lectures on quantum mechanics. Tomonaga Shin'ichirō (朝永振一郎) and Yukawa Hideki (湯川秀樹), then young unpaid assistants at Kyōto, attend these lectures.

1932. Tomonaga becomes a member of Nishina's lab at RIKEN. Meanwhile, Yukawa is appointed lecturer at the Kyōto university and starts working on nuclear theory.

1933. Yukawa and Tomonaga attend the annual meeting of the Physico-Mathematical Society of Japan held in Sendai. An interplay between the two on nuclear forces follows.

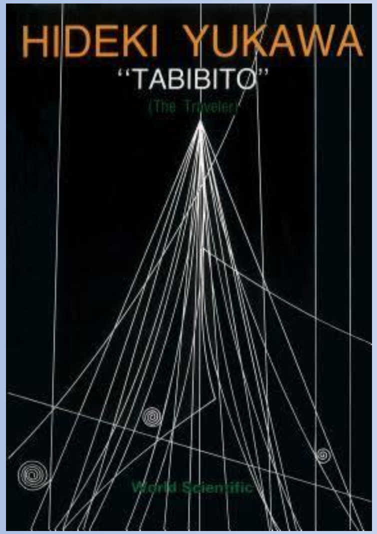
1933-34. Nishina and Tomonaga perform calculations about the collision of neutrons with matter and on the range of neutron-proton interactions.

1934. Yukawa moves to Ōsaka and writes his first fundamental paper on meson theory.

1937. Bohr visits Japan and criticizes Yukawa's introduction of a new elementary particle. Tomonaga goes to Leipzig and stays there until 1939. In that year, also Yukawa visits Europe. Both Tomonaga and Yukawa visited Europe only after meson theory appeared.

Yukawa and the nuclear force

In 1932, Yukawa translated Heisenberg's papers in Japanese and writes a short introduction to them. This marked the beginning of his interest in the problem of the nuclear force. It took him two years to conceive meson theory. His recollections vividly describe his struggles:



'The problem that I focused on was that the nature of the forces that act upon the neutrons and protons making up the nucleus — that is, the nature of the nuclear forces. By confronting this difficult problem, I committed myself to long days of suffering. In fact, the period from the autumn of the seventh year of Shōwa to the autumn of the ninth year (1932 to 1934) were the most difficult years of my life. The fact that I was suffering, however, was very satisfying to me; I felt like a traveler carrying a heavy burden and struggling up a slope. I experienced the sorrows and joys of a scientist'

(Yukawa, Tabibito, p. 312)

H. Yukawa, Tabibito, World Scientific 1982

Yukawa and the nuclear force

The steps he took in those two years are documented by the talk he gave in 1933 in Sendai, and by the related notes which are recorded in archival material. There, he tried to derive the form of the p-n interaction from the relativistic quantum mechanics of the electron, in analogy with quantum electrodynamics. In the abstract, he claimed:

'We can consider […] that by analogy with radiation (in the same sense that the radiation is the mediator of interactions between electrons, protons and other charged particles), the electron is the mediator of the interaction between proton and neutron and acts like a kind of field in the inside of nuclei. Then, we may solve the above-mentioned equation for the electron to find the form of interaction between neutron and proton. From the fact that the electron has the finite rest mass, we may expect that the interaction energy would decrease rapidly as the distance between neutron and proton becomes large in comparison with $h/(2\pi mc)$.'

H. Yukawa, Bull. Phys. Math. Soc. Japan, 1933, translated in R.Kawabe, Progr. Theor. Phys. Suppl. 1991, p.247

L. M. Brown, Prog. Theor. Phys. Suppl. 1985

Yukawa and the nuclear force

In the talk, he states that:

'If we consider the neutron as an elementary particle, the interaction mediated by the electromagnetic field does not exist between protons and neutrons and between one neutron and another. In this case, the interaction due to the "Platzwechsel der Ladung" *proposed by Heisenberg can be interpreted as follows: The very fact that a neutron emits an electron and changes into a proton, and that a proton absorbs an electron and turns into a neutron, is the cause of the interaction between neutron and proton, just as the emission and absorption of radiation is the cause of the interaction between one electron and another. The electron can be considered as the mediator of the interaction between proton and neutron or between one neutron and another, just as the radiation is the mediator of the interaction between one charged particle and another.'*

H. Yukawa, document no. YHAL E05 080 U01, translated in R.Kawabe, Progr. Theor. Phys. Suppl. 1991, p.248

Yukawa and the nuclear force

Two unpublished documents of that period give valuable information on Yukawa's reasoning, as well as some incomplete calculations. Interestingly, in the first of these documents, before turning to his proposal, Yukawa briefly admits that a phenomenological approach would be more effective:

' [W]hen we regard the neutron as an elementary particle, there immediately arises the question: What kind of force acts between the neutron and the charged particles? To solve this problem the most effective method at present might be to calculate the scattering of neutrons on nuclei by assuming a suitable interaction between neutrons and nuclei and comparing the results with experiment. In effect, one can assume a force decreasing rapidly with the distance (Massey, Proc. R. Soc. London 138 (1933), 460) [...] in any case we cannot justify the assumptions.'

As Kawabe points out, Massey's cited paper followed another one (Proc. R. Soc. Lon, 137, 1932) in which the following potential appears:

$$V(r) = -\frac{Ae^{-\mu r}}{r}$$

This may have been known by Yukawa, but he does not refer to it.

H. Yukawa, The Roles of the Electron for Nuclear Structure, document no. YHAL E05 060 U01,translated in R.Kawabe, Progr. Theor. Phys. Suppl. 1991, p.250-252; The Bose electron, document no. YHAL F01 010 U01, translated in R.Kawabe, Progr. Theor. Phys. Suppl. 1991, p.253-257.

Yukawa and the nuclear force

In the end, Yukawa's program does not work, as he admits in the talk:

'In any case, the practical calculation does not yield the looked-for result that the interaction term decreases rapidly as the distance becomes larger than $h/(2\pi mc)$ *unlike what I wrote in the abstract of this talk.'*

H. Yukawa, document no. YHAL E05 080 U01, translated in R.Kawabe, Progr. Theor. Phys. Suppl. 1991, p.249

Due to the difficulties he met, which he attributed to the incompleteness of relativistic quantum mechanics, Yukawa briefly turned to foundational issues in this subject. This will be the subject of his 1934 talk in Sendai – where he introduced his famous *maru* – but, in fact, his 1933 talk already starts with the words:

'The problems of atomic nuclei, especially the problem of electrons in the nucleus, might not be solved until we reflect on the foundation of quantum mechanics and complete the correct relativistic quantum theory. It cannot be solved only by partial and formal modification'

H. Yukawa, document no. YHAL E05 080 U01, translated in R.Kawabe, Progr. Theor. Phys. Suppl. 1991, p.248

Yukawa and the nuclear force

In 1934 Yukawa learnt about the work by Fermi and turned again to the nucleus. After considering himself nuclear forces as due to neutrino-electron exchange, and having later read about the negative results of this approach, he finally changed line of attack, and proposed *a brand new field*, tailored on the known properties of the interaction:

'I was heartened by the negative result, and it opened my eyes, so that I thought: Let me not look for the particle that belongs to the field of the nuclear force among the known particles, including the new neutrino. If I focus on the characteristics of the nuclear force field, then the characteristics of the particle I seek will become apparent. When I began to think in this manner, I had almost reached my goal'

(Yukawa, Tabibito, p. 201)

L. M. Brown, Centaurus 1981

Yukawa and the nuclear force

Yukawa seeked a field theory for the nuclear force and tried to infer its properties from the known properties of the latter. He assumed that the potential had the simple form $\pm g^2 \frac{e^{-\lambda r}}{r}$, which could be viewed as the static, spherically symmetric solution of the equation:

$$\left(\Delta - \frac{1}{c^2}\frac{\partial^2}{\partial t^2} - \lambda^2\right) U = 0$$

Then he retraced the calculations he performed in 1933 for the case of the electron, with this equation in place of the Dirac equation, and assuming an appropriate source,

$$\left(\Delta - \frac{1}{c^2}\frac{\partial^2}{\partial t^2} - \lambda^2\right) U = -4\pi g \tilde{\psi} \left(\frac{\rho^\xi - i\rho^\eta}{2}\right) \psi$$

H. Yukawa, Proc. Phys. Math. Japan 1935

Yukawa and the nuclear force

He then solved this equation and completed his program of writing down the Hamiltonian for the n-p system. This turns out to be essentially equivalent to Heisenberg's one, with the above potential, *with the minus sign*, playing the role of the exchange integral *J(r)*.

$$H = \frac{\mathbf{p}_1^2}{2M} + \frac{\mathbf{p}_2^2}{2M} + \frac{g^2}{2}\left(\rho_1^\xi \rho_2^\xi + \rho_1^\eta \rho_2^\eta\right) \frac{e^{-\lambda r_{12}}}{r_{12}} + \left(\rho_1^\zeta + \rho_2^\zeta\right) D$$

'The Hamiltonian is equivalent to Heisenberg's Hamiltonian, if we take for 'Platzwechselintegral' $-g^2 \frac{e^{-\lambda r}}{r}$*, except that the interaction between the neutrons and the electrostatic repulsion between the protons are not taken into account. Heisenberg took the positive sign for* $J(r)$*, so that the spin of the lowest energy state of* H^2 *was 0, whereas in our case, owing to the negative sign in front of* g^2*, the lowest energy state has the spin 1, which is required from the experiment.'*

(Yukawa 1934 p. 51)

H. Yukawa, Proc. Phys. Math. Japan 1935

Yukawa and the nuclear force

Yukawa then noticed that quantum field theory implied that this new field was associated to massive quanta, with mass proportional to the parameter λ. Hence, *the range of the force is related to the mass of the quanta*, which turned out to be about 200 electron masses. He therefore called these *heavy* quanta (as opposed to *light* quanta). This result was later interpreted by G. C. Wick as a consequence of the uncertainty relations.

'Now, such an interaction […] can be described by means of a field of force, just as the interaction between the charged particles is described by the electromagnetic field. In the quantum theory this field should be accompanied by a new sort of quantum, just as the electromagnetic field is accompanied by the photon'

(Yukawa 1934 p. 48*)*

H. Yukawa, Proc. Phys. Math. Japan 1935
G. C. Wick. Nature, 1938

Some comments

- Mesons are unobservable since the energies relevant for ordinary nuclear transmutations are so low that the meson waves are damped, and cannot exit the nucleus (today we say that the meson is a *virtual* particle). Yukawa suggested looking for them in cosmic rays.
- Yukawa also assumed that the meson coupled – with a *different* coupling constant g' – to light particles (electrons and neutrinos), so to include Fermi interactions. He obtained G_F=$4\pi g g'/\lambda^2$ with $g' \approx 10^{-8} g$. He was the first to clearly distinguish between these two interactions.
- Yukawa's U-field should have been the analog of the electromagnetic scalar potential. In the 1934 paper, he did not consider the analog of the vector potential because at the time there were no known relativistic equations for massive vector fields.
- Yukawa came back to meson theory in 1937, after the discovery of a particle apparently compatible with his heavy quanta in cosmic rays. This event also marks the beginning of the interest of western physicists in his theory. With his group, Yukawa then began a systematic development of the quantum field theory of mesons.

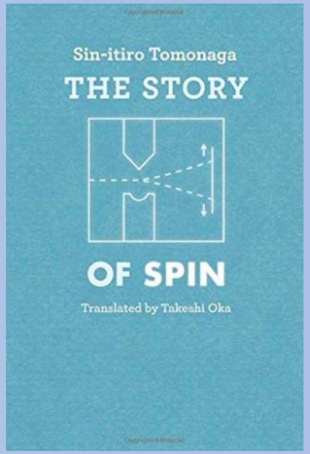
Tomonaga’s contributions

In his paper, Yukawa explicitly acknowledges Tomonaga’s contributions:

‘Using the Hamiltonian (10) for heavy particles, we can calculate the mass defect of H^2 *and the probability of scattering of a neutron by a proton’*

‘These calculations were made previously, according to the theory of Heisenberg, by Mr. Tomonaga, to whom the writer owes much. A little modification is necessary in our case.’

(Yukawa 1934, p. 52)



Tomonaga’s role can be reconstructed from archival documents (Yukawa archives, YITP) and from Tomonaga’s recollections (The story of spin, Lect. 12).

https://www-yukawa.phys.sci.osaka-u.ac.jp/en/archive/387

S. Tomonaga, The Story of Spin, Transl. by T. Oka, U. Chicago Press 1997

Tomonaga's contributions

In April 1933, at Sendai, Nishina and Tomonaga report on their calculations. The abstract of their talk reads:

'We have analyzed scattering of neutron by proton using Heisenberg's theory on nuclear structure, assuming the shape of interactions between neutron and proton, and taking into account the mass defect of hydrogen 2. Our result has been compared with experimental results.'

At Yukawa's request, Tomonaga wrote him a 7 page letter, in which he gave details of his calculations involving several short-range phenomenological potentials. In particular, he mentions the expression $J(r) = Ae^{-\lambda r}/r$.

Tomonaga's letter:
https://www-yukawa.phys.sci.osaka-u.ac.jp/en/archive/387

M. Konuma et al. Proc. 12° Asia Pacific Physics Conf.
R.Kawabe, Progr. Theor. Phys. Suppl. 1991
T. Yamazaki, Nucl. Phys. A 2008

Conclusions

- The 1930s were a decade of impressive development for theoretical nuclear physics. Fermi's and Yukawa's theories are the first quantum field theories other than quantum electrodynamics.
- In the 1930s, Japanese physicists such as Nishina were a consolidated presence in European university, and hence could absorb the new developments, contribute to them and bring them to their country.
- The decisive steps towards a quantum field theory of the nuclear interaction and the prediction of the meson were made by the new generation of Japanese physicists.
- Yukawa's assumptions were that the principles of quantum theory were valid inside the nucleus, and that the nuclear interaction is a fundamental force analogous to electromagnetism.
- A fruitful interplay between Yukawa and Tomonaga in 1933-34 was important for Yukawa's theory. Later, Tomonaga worked extensively on meson theory, developing his intermediate coupling theory.
- The concepts of a particle as the mediator of the interaction, of virtual particle, of the relation of the mass of the mediator with the range of the interaction were not well-established at the time, reaching maturity only after WWII (Carson, 1996); Yukawa's theory was in fact a decisive step.
- Later, Yukawa's proposals for a divergence-free QED (Yukawa's *maru*) inspired Tomonaga in the development of his super-many time theory. But that is another story.
- Nuclear interactions are in fact much more complicated than either of our main characters could envision. The story we told is just the beginning.

THANK YOU FOR YOUR ATTENTION!