Backreaction of scalar radiation on black holes

Marco de Cesare

Scuola Superiore Meridionale, Napoli, Italy

INFN, Sezione di Napoli, Italy

talk based on

MdC, R.Oliveri, arXiv:2305.04970 [gr-qc]

Phys. Rev. D 108 (2023) 044050

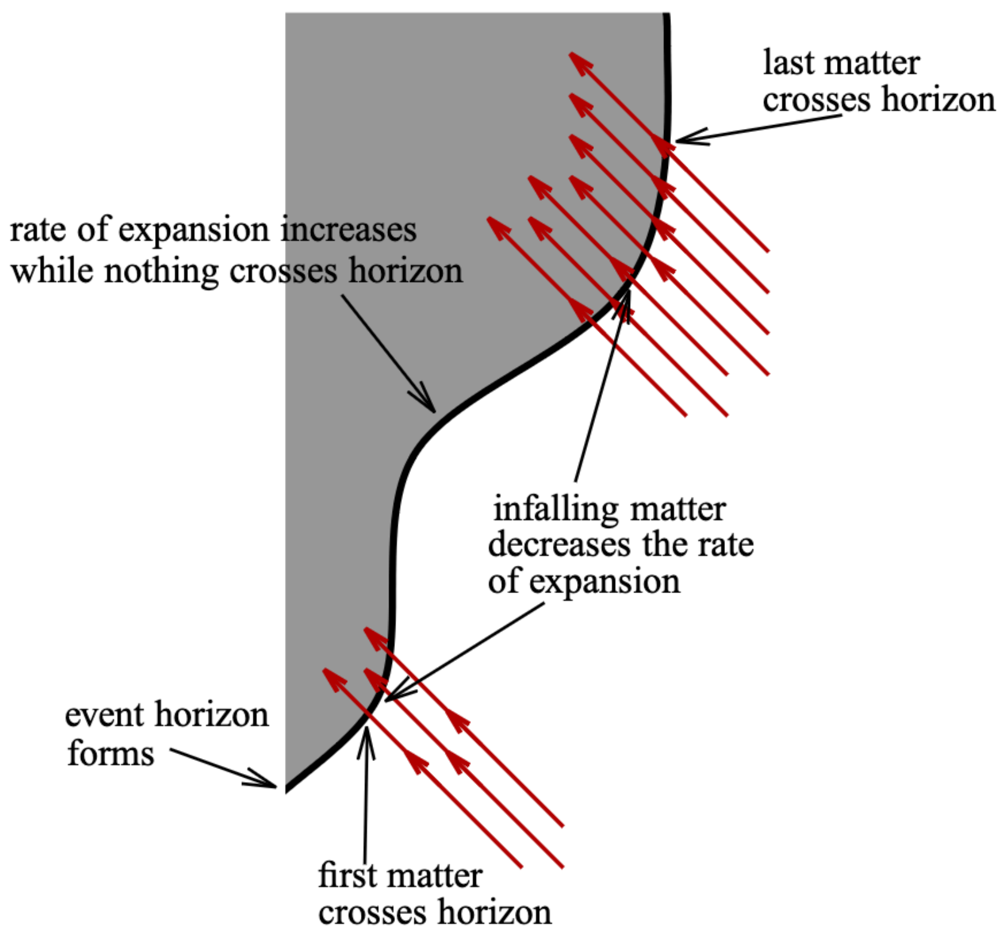
Re[incontri] di Fisica Partenopea, Università di Napoli "Federico II"

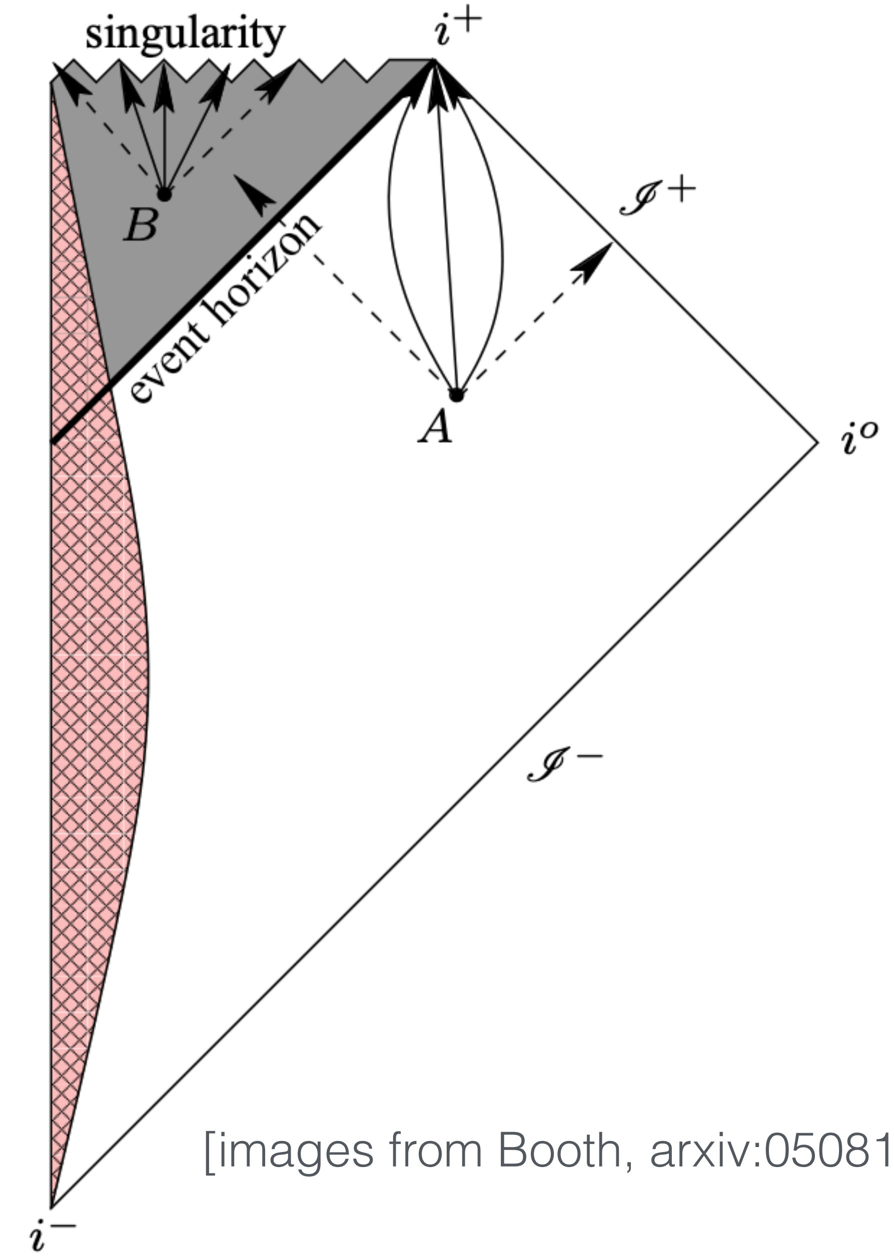
December 19th 2023

Introduction

The event horizon of a black hole is the boundary of a spacetime region whence no signal can ever escape (boundary of the past of $\mathscr{I}^+$)

The area of the event horizon of a BH cannot decrease (black hole area theorem) [Hawking (1971)]





[images from Booth, arxiv:0508107]

However, the event horizon is a teleological object, describing a global property of spacetime and thus *cannot* be probed by local measurements

Introduction

Different (quasi-local) definitions of a black hole horizon have been proposed that are better suited in dynamical situations [Hayward; Ashtekar, Krishnan; Booth, Fairhurst]

The evolution of such quasi-local horizons is causal and determined by the ingoing flux of energy-momentum (and gravitational waves)

In this talk, I will adopt Hayward's definition of the BH horizon as the *future outer trapping horizon* (FOTH) $\theta_l = 0$, $\theta_n < 0$, $\pounds_n \theta_l < 0$
(this is a 3-surface foliated by marginally trapped surfaces)

Question: Can we compute how the horizon evolves for given initial data, assigned in a region asymptotically far from the black hole?

The goal is to understand how the evolution of matter on large scales influences local systems such as black holes

Introduction

Previous work focused on (this is just a partial list):

- exact solutions of GR [McVittie; Einstein,Straus; Husain,Martinez,Nuñez],
- test-fluid approximation [Bondi,Hoyle,Lyttleton; Cruz-Osorio,Rezzolla,Lora-Clavijo,Font,Herdeiro,Radu],
- steady-state accretion for generic matter fields [Babichev, Dokuchaev, Eroshenko],
- numerical studies of absorption [Crispino,Dolan,Leite,Macedo,Oliveira; Guzman,Lora-Clavijo],
- generalized McVittie in modified gravity (Horndesky) [Miranda, Vernieri, Capozziello, Faraoni]

Here we adopt a different approach, solving the Einstein equations perturbatively (without assuming steady state and retaining all relativistic effects)

our goal: Compute the evolution of the horizon as a backreaction effect

Scalar perturbations on Schwarzschild

Take a general spherically symmetric metric (in Schwarzschild coordinates)

$$ds^2 = -A(t,r)e^{\nu(t,r)}dt^2 + A^{-1}(t,r)dr^2 + r^2 d\Omega^2$$

$$A(t,r) = 1 - 2m(t,r)/r$$

Misner-Sharp mass

Spherical perturbations around a Schwarzschild background

$$m(t,r) = M + m^{(1)}(t,r) + m^{(2)}(t,r) + \dots$$

$$\nu(t,r) = 0 + \nu^{(1)}(t,r) + \nu^{(2)}(t,r) + \dots$$

$$\phi(t,r) = 0 + \phi^{(1)}(t,r) + \dots$$

Assume a minimally-coupled massless scalar

$$T_{ab} = \partial_a\phi\partial_b\phi - \frac{1}{2}g_{ab}\left(g^{cd}\partial_c\phi\partial_d\phi\right)$$

Perturbative equations

$$G^a_{\ b} = 8\pi T^a_{\ b}$$

First order

$$\boxed{\Box\phi = 0}$$

$$dm^{(1)} = d\nu^{(1)} = 0 \Longrightarrow$$ can be reabsorbed in the background

Second order

take the (t,t) and (t,r) components of the field eqs.

$$\frac{\partial m^{(2)}}{\partial r} = 2\pi r^2 \left[\left(1 - \frac{2M}{r}\right)^{-1} \left(\frac{\partial \phi}{\partial t}\right)^2 + \left(1 - \frac{2M}{r}\right) \left(\frac{\partial \phi}{\partial r}\right)^2 \right]$$

$$\frac{\partial m^{(2)}}{\partial t} = 4\pi r^2 \left(1 - \frac{2M}{r}\right) \frac{\partial \phi}{\partial t} \frac{\partial \phi}{\partial r}$$

Backreaction in the near-horizon region

Introduce the RW tortoise coordinate $r_* = r + 2M \log|r/(2M) - 1|$
and take the limit $r_* \to -\infty$

$$\frac{\partial m^{(2)}}{\partial r_*} \approx 2\pi \left[\left(\frac{\partial (r\phi)}{\partial t} \right)^2 + \left(\frac{\partial (r\phi)}{\partial r_*} \right)^2 \right] ,$$

$$\frac{\partial m^{(2)}}{\partial t} \approx 4\pi \frac{\partial (r\phi)}{\partial t} \frac{\partial (r\phi)}{\partial r_*} .$$

Change to null coordinates $u = t - r_*$, $v = t + r_*$

$$\boxed{\frac{\partial m^{(2)}}{\partial v} \approx 4\pi \left(\frac{\partial (r\phi)}{\partial v} \right)^2} \qquad \frac{\partial m^{(2)}}{\partial u} \approx -4\pi \left(\frac{\partial (r\phi)}{\partial u} \right)^2 = 0$$

equation for the backreaction on the mass

for purely ingoing radiation
at the horizon

Scattering solutions to the wave equation

[Starobinski; Ford; Page; Unruh; Sanchez; ...]   monograph: [Futterman, Handler, Matzner]

wave equation: $\frac{\partial^2 \phi}{\partial t^2} - \frac{\Delta}{r^4}\frac{\partial}{\partial r}\left(\Delta \frac{\partial \phi}{\partial r}\right) = 0$ $\Delta \equiv r^2 - 2Mr$

going to Fourier space $\phi(t,r) = (2\pi)^{-1} \int d\omega \, R(\omega, r) e^{-i\omega t}$ and changing variables $x \equiv r/(2M) - 1$

$$\boxed{x^2(1+x)^2 \frac{d^2 R}{dx^2} + x(1+x)(1+2x)\frac{dR}{dx} + (2M\omega)^2 (1+x)^4 R = 0}$$

scattering boundary conditions, ingoing modes at the horizon

$R \sim T(\omega)\, x^{-i(2M\omega)}$ as $x \to 0$

$R \sim \mathcal{I}(\omega) e^{-i(2M\omega)x}/x + \mathcal{R}(\omega) e^{i(2M\omega)x}/x$ as $x \to +\infty$

low-frequency approximation: $\epsilon \equiv 2M\omega$, such that $|\epsilon| \ll 1$

Scattering solutions to the wave equation

matched asymptotics

$$x^2(1+x)^2\frac{d^2R}{dx^2} + x(1+x)(1+2x)\frac{dR}{dx} + \epsilon^2(1+x)^4 R = 0$$

Near region $x|\epsilon| \ll 1$

$$x^2(1+x)^2\frac{d^2R}{dx^2} + x(1+x)(1+2x)\frac{dR}{dx} + \epsilon^2(1+4x) R = 0$$

$$R_{\rm near}(x) = a\, x^{-i\epsilon}(1+x)^{\sqrt{3}\epsilon}\ {}_2F_1\left(1+(\sqrt{3}-i)\epsilon, (\sqrt{3}-i)\epsilon; 1-2i\epsilon; -x\right)$$

large-x limit: $R_{\rm near}(x) \approx \frac{\mathcal{T}}{2M}\left(1+\frac{i\epsilon}{x}\right)$

Far region $x \gg 1$

$$\frac{d^2R}{dx^2} + \frac{2}{x}\frac{dR}{dx} + \epsilon^2\left(1+\frac{2}{x}\right)R = 0$$

$$R_{\rm far}(x) = e^{-i|\epsilon|x}\left(c_1\ U(1+i|\epsilon|, 2, 2i|\epsilon|x) + c_2\ {}_1F_1(1+i|\epsilon|; 2; 2i|\epsilon|x)\right)$$

small $x|\epsilon|$ limit: $R_{\rm far}(x) \approx \frac{1}{2M}\left(i\epsilon\left(\mathcal{R}-\mathcal{I}\right) + \frac{\left(\mathcal{R}+\mathcal{I}\right)\left(1+i\gamma|\epsilon|\right)}{x}\right)$

Scattering solutions to the wave equation

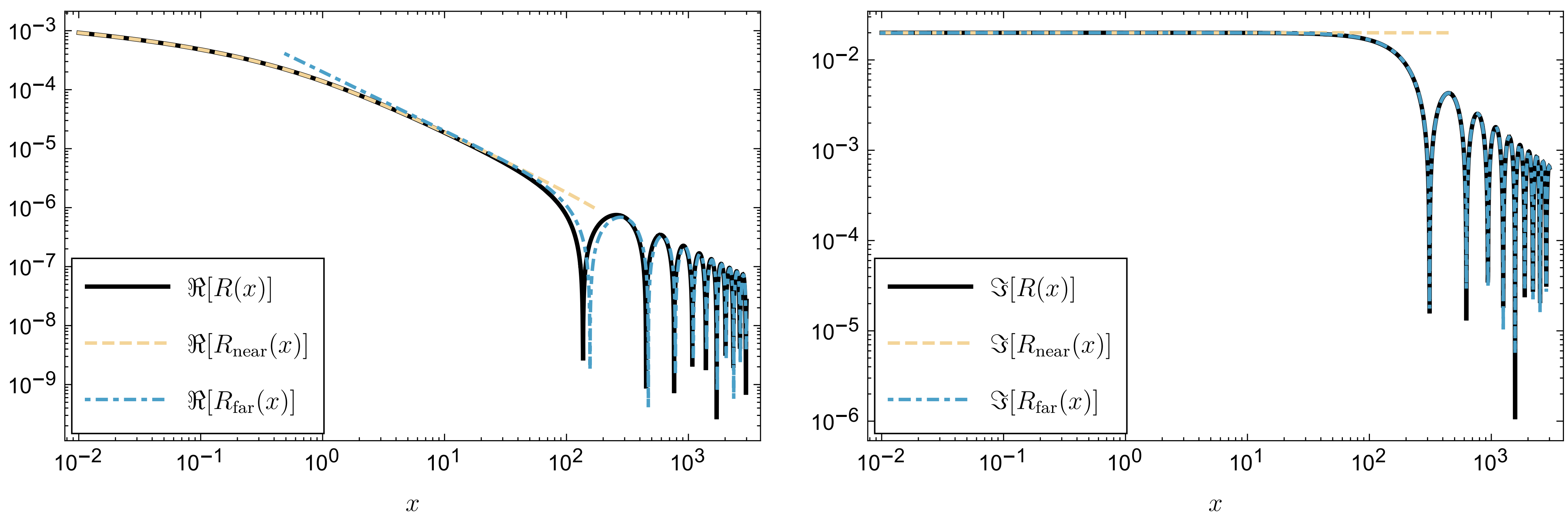
matched asymptotics

Matching the two asymptotics we obtain

$$\mathscr{T} = -2i\epsilon\,\mathscr{I} + \mathcal{O}(\epsilon^3)\,, \quad \mathscr{R} = \left(-1 + 2\epsilon^2\right)\mathscr{I} + \mathcal{O}(\epsilon^3)$$

that satisfy $|\mathscr{T}|^2 + |\mathscr{R}|^2 \simeq |\mathscr{I}|^2$

comparison with numerics for $\epsilon = 10^{-2}$ (with $\mathscr{I} = 1$, $2M = 1$)



Scattering of wave packets and backreaction

Transform back to real space and use our result for $\mathcal{T}(\omega)$
to evolve the wave packet past the potential barrier

$$\phi_{\mathrm{H,in}}(v,r) = \frac{1}{2\pi r}\int_{-\infty}^{+\infty} d\omega\,(-2i)(2M\omega)\mathcal{I}(\omega)e^{-i\omega v} = \frac{4M}{r}\frac{d}{dv}\left(r\phi_{\mathrm{in}}(v,r)\right)$$

Combine this result with the backreaction equations

$$\frac{\partial m^{(2)}}{\partial v} \approx 4\pi\left(\frac{\partial(r\phi)}{\partial v}\right)^2 \qquad \frac{\partial m^{(2)}}{\partial u} \approx 0 \implies m^{(2)} \approx m^{(2)}(v)$$

near the horizon

Mass of the perturbed BH: $\overline{M}(v) = M + \Delta M(v)$, with $\Delta M(v) \equiv \lim_{u\to+\infty} m^{(2)}(u,v)$

$$\boxed{\frac{d(\Delta M)}{dv} \approx 64\pi M^2\left(\frac{d^2\left(r\phi_{\mathrm{in}}\right)}{dv^2}\right)^2}$$

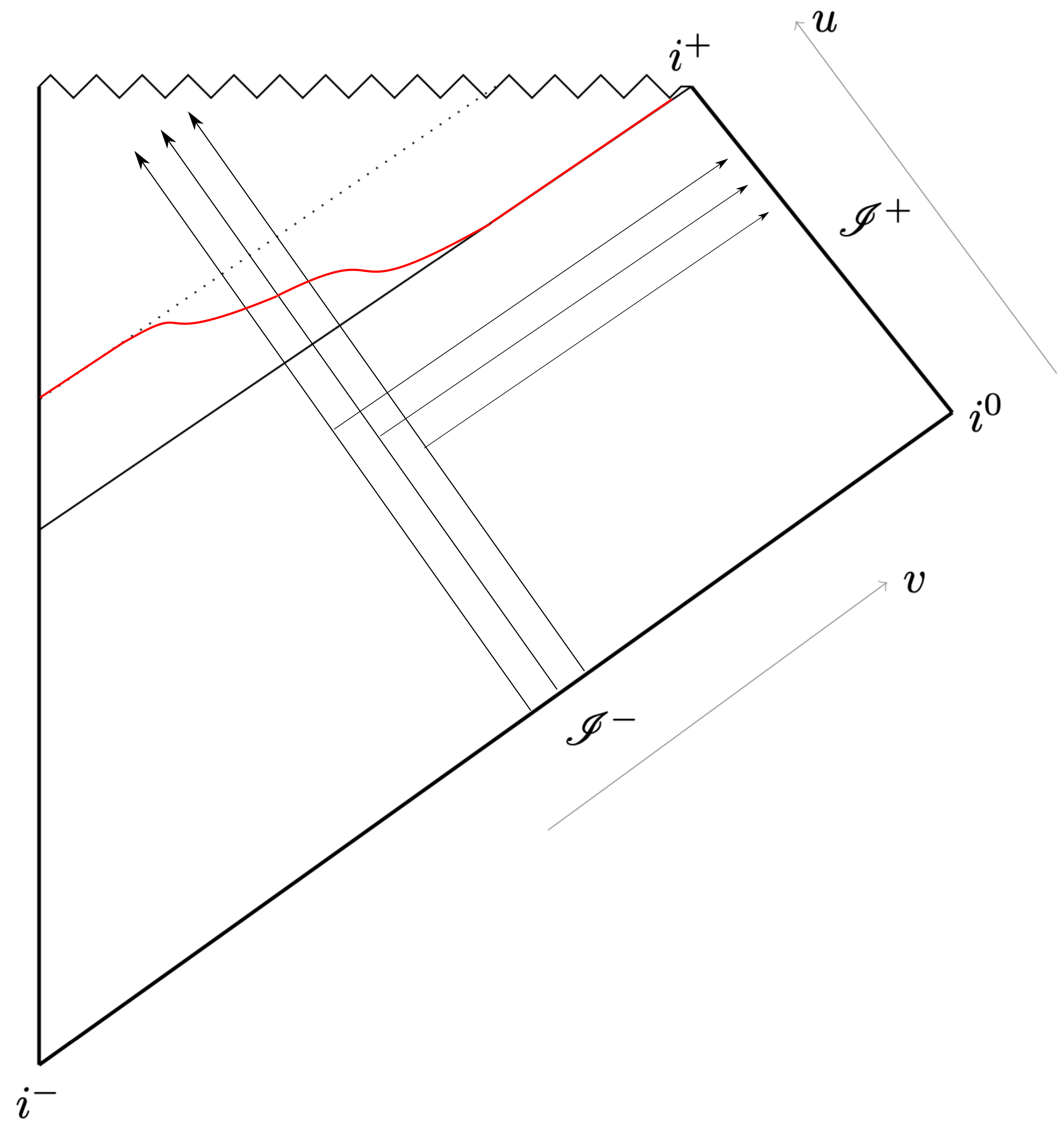
BH mass growth rate

data on $\mathscr{I}^-$

Evolution of the trapping horizon

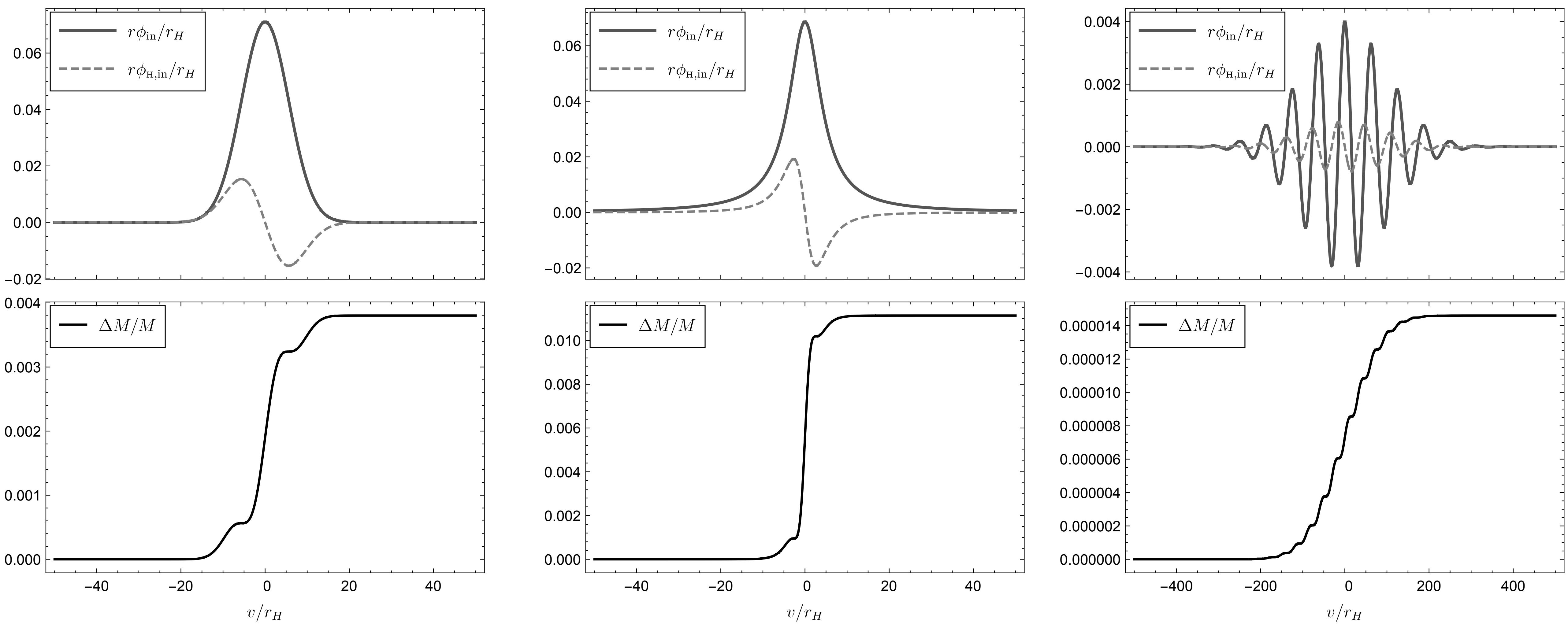
Mass growth = expansion of the trapping horizon

(they are proportional in spherical symmetry)



Application 1: coherent radiation

wave packets with assigned profile



$\phi_{\rm in}(v,r) = \dfrac{A}{\sqrt{2\pi}\sigma r}\exp\left(-\dfrac{(v-v_0)^2}{2\sigma^2}\right)$	$\phi_{\rm in}(v,r) = \dfrac{A}{\pi r}\dfrac{\gamma}{\gamma^2+(v-v_0)^2}$	$\phi_{\rm in}(v,r) = \dfrac{A}{\sqrt{2\pi}\sigma r}\exp\left(-\dfrac{(v-v_0)^2}{2\sigma^2}\right)\cos\left(\omega(v-v_0)\right)$
$A = r_H^2,\ \ \sigma \simeq 5.617\, r_H,\ \ \gamma \simeq 4.642\, r_H$ (same ingoing flux and normaliz.)		$A = r_H^2,\ \ \omega = 10^{-1}/r_H,\ \ \sigma = 10^2\, r_H$

Application 2: incoherent radiation

amplitude: $\mathcal{I}(\omega) = A(\omega)e^{-i\varphi_\omega}$

random phase model
$\langle e^{-i\varphi_\omega} e^{i\varphi_{\omega'}} \rangle = 2\pi C\,\delta(\omega - \omega')$

ingoing energy flux: $\mathcal{F}_{\rm in}(v) \equiv 4\pi r^2 \left(\frac{\partial}{\partial v}\phi_{\rm in}(v,r) \right)^2$

$$\langle \mathcal{F}_{\rm in}(v) \rangle = 2C \int_{-\infty}^{+\infty} d\omega\, A(\omega)A(-\omega)\omega^2$$

$$\left\langle \frac{d(\Delta M)}{dv} \right\rangle \approx 16 \left(\frac{\int_{-\infty}^{+\infty} d\omega\, A(\omega)A(-\omega)\omega^4}{\int_{-\infty}^{+\infty} d\omega\, A(\omega)A(-\omega)\omega^2} \right) M^2 \langle \mathcal{F}_{\rm in} \rangle$$ = constant

in a Gaussian model $A(\omega) = \mathcal{A} \exp(-(\omega - \bar{\omega})^2/(2\sigma^2))$:

$$\left\langle \frac{d(\Delta M)}{dv} \right\rangle \approx 24\sigma^2 M^2 \langle \mathcal{F}_{\rm in} \rangle$$

Summary of the main results

- The evolution of the BH mass / trapping horizon can be computed in second-order perturbation theory as a backreaction effect
- In the low-frequency limit we can derive a closed-form expression for the accretion rate in terms of initial data on $\mathscr{I}^-$
- Applications: wave packets; incoherent radiation

Future developments / Outlook

- How does the accretion rate depend on the scalar field potential and on the spacetime asymptotics (e.g., cosmological spacetimes)? Generalization to different matter fields.
- What is the impact of the black hole spin on accretion?
- Possible connections with works on BHs in (scalar) dark matter clouds
  [Annulli, Cardoso, Clough, Ferreira, Hui, Ikeda, Vicente, Zilhão,…]